Overview

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  NFA-representation for the bad prefixes

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- regular safety properties:
  NFA-representation for the bad prefixes
- other regular LT properties:
  representation by ω-automata, i.e., acceptors for infinite words

Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

- regular safety properties ⟵
- ω-regular properties
- model checking with Büchi automata

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

Let E be a LT property over AP, i.e., $E \subseteq (2^{AP})^\omega$.

E is called a safety property if for all words

$$\sigma = A_0 A_1 A_2 \ldots \in (2^{AP})^\omega \setminus E$$

there exists a finite prefix $A_0 A_1 \ldots A_n$ of σ such that *none* of the words $A_0 A_1 \ldots A_n B_{n+1} B_{n+2} B_{n+3} \ldots$ belongs to E, i.e.,

$$E \cap \{\sigma' \in (2^{AP})^\omega : A_0 \ldots A_n \text{ is a prefix of } \sigma'\} = \varnothing$$

Such words $A_0 A_1 \ldots A_n$ are called bad prefixes for E.

$$BadPref \stackrel{\text{def}}{=} \text{set of bad prefixes for } E \subseteq (2^{AP})^+$$

Regular safety properties

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E is called regular iff the language

$BadPref$ = set of all bad prefixes for E

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$$BadPref = \mathcal{L}(\mathcal{A}) \text{ for some NFA } \mathcal{A}$$
over the alphabet 2^{AP}

is regular.

Nondeterministic finite automata (NFA)

IS2.5-15

Nondeterministic finite automata (NFA)

NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$

- Q finite set of states
- Σ alphabet
- $\delta : Q \times \Sigma \to 2^Q$ transition relation
- $Q_0 \subseteq Q$ set of initial states
- $F \subseteq Q$ set of final states, also called accept states

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run for a word $A_0 A_1 \ldots A_{n-1} \in \Sigma^*$:

state sequence $\pi = q_0\, q_1 \ldots q_n$ where $q_0 \in Q_0$
and $q_{i+1} \in \delta(q_i, A_i)$ for $0 \leq i < n$

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run π is called accepting if $q_n \in F$

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$\mathcal{L}(\mathcal{A})$ = set of finite words over Σ that have an accepting run in $\mathcal{A}$

Nondeterministic finite automata (NFA)

NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$

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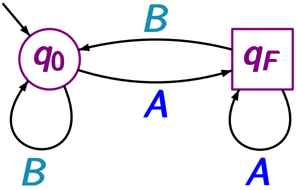
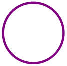
Notations in pictures for NFA



 initial state

 nonfinal state

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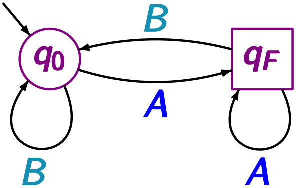
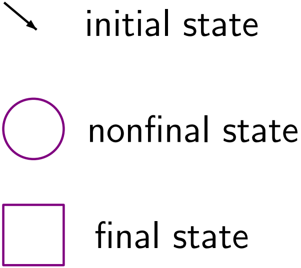




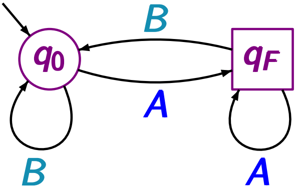
NFA $\mathcal{A}$ with state space $\{q_0, q_F\}$

q_0 initial state

q_F final state

alphabet $\Sigma = \{A, B\}$

Notations in pictures for NFA



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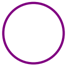 nonfinal state

 final state

accepted language $\mathcal{L}(\mathcal{A})$:

set of all finite words over $\{A, B\}$
ending with letter A

Symbolic notations

for transitions in **NFA** over the alphabet $\Sigma = 2^{AP}$

Symbolic notations

NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ over the alphabet $\Sigma = 2^{AP}$

symbolic notation for the labels of transitions:

If Φ is a propositional formula over AP then

$q \xrightarrow{\Phi} p$ stands for the set of transitions $q \xrightarrow{A} p$

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Example: if $AP = \{a, b, c\}$ then

$$q \xrightarrow{a \wedge \neg b} p \;\widehat{=}\; \{\, q \xrightarrow{A} p : A{=}\{a, c\} \text{ or } A{=}\{a\} \,\}$$

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$$q \xrightarrow{a \wedge \neg b} p \;\widehat{=}\; \left\{ q \xrightarrow{A} p : A=\{a, c\} \text{ or } A=\{a\} \right\}$$

$$q \xrightarrow{true} p \;\widehat{=}\; \left\{ q \xrightarrow{A} p : A \subseteq AP \right\}$$

Symbolic notations

$$\Sigma = \{A, B\}$$

$$q \xrightarrow{A} p$$

$$q \xrightarrow{B} p$$

$$AP = \{a, b\}$$

$$\Sigma = 2^{AP}$$

$$q \xrightarrow{\{\}} p$$

$$q \xrightarrow{\{a\}} p \qquad q \xrightarrow{\{b\}} p$$

$$q \xrightarrow{\{a, b\}} p$$

$$q \xrightarrow{a} p \ \widehat{=} \left\{ q \xrightarrow{\{a\}} p \quad q \xrightarrow{\{a, b\}} p \right\}$$

Regular safety properties

A safety property $E \subseteq (2^{AP})^\omega$ is called regular iff

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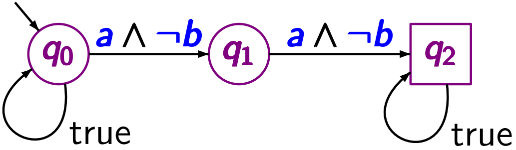

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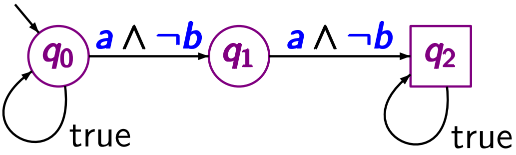

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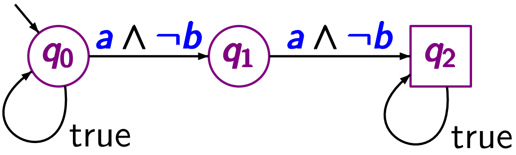

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symbolic notation:

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safety property E: "$a \wedge \neg b$ never holds twice in a row"

Example: regular safety property

"Every red phase is preceded by a yellow phase"

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set of all infinite words $A_0\, A_1\, A_2 \ldots$ s.t. for all $i \geq 0$:

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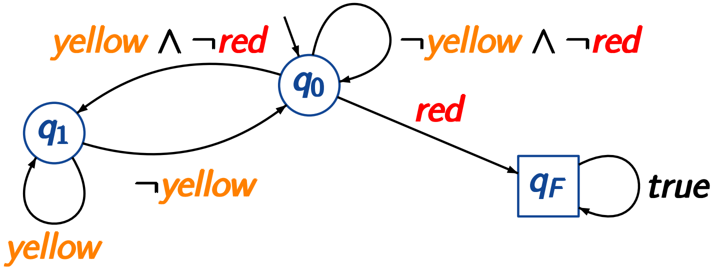
DFA for all (possibly non-minimal) bad prefixes



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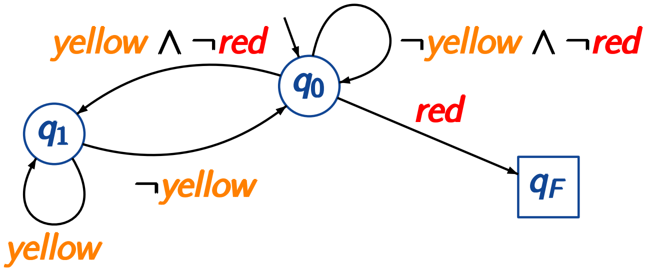

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Correct or wrong?

Every invariant is regular.

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Let E be an invariant with invariant condition Φ

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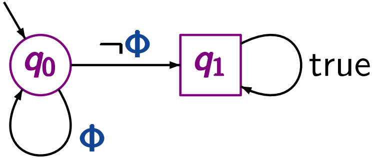

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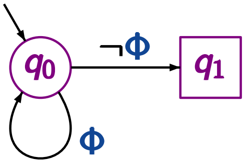

is a DFA for the language of all minimal bad prefixes

Example: DFA for *MUTEX*

"The two processes are never simultaneously
in their critical sections"

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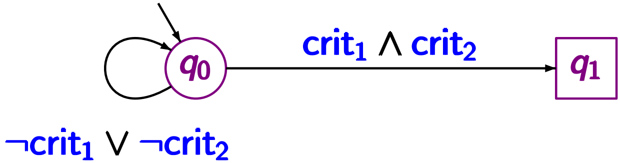
DFA for minimal bad prefixes over the alphabet 2^{AP} where $AP = \{crit_1, crit_2\}$



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- E is a safety property, but
- the language of (minimal) bad prefixes is *not* regular

Verifying regular safety properties

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given: finite TS $\mathcal{T}$
regular safety property E
(represented by an **NFA** for its bad prefixes)

question: does $\mathcal{T} \models E$ hold **?**

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method: relies on an analogy between the tasks:

- checking language inclusion for **NFA**
- model checking regular safety properties

language inclusion for NFA	verification of regular safety properties
$\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ **?**	$Traces(\mathcal{T}) \subseteq E$ **?**

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$\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ **?**	$Traces(\mathcal{T}) \subseteq E$ **?**
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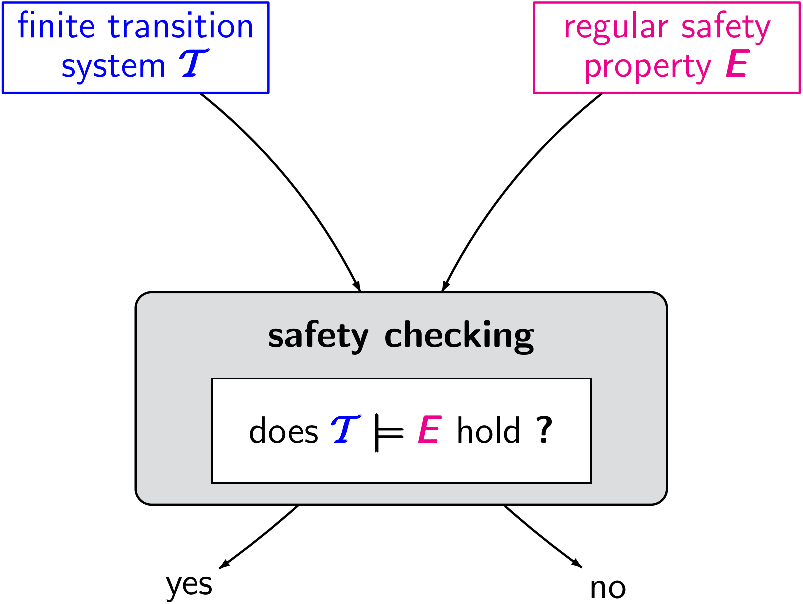
2. construct NFA $\mathcal{A}$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\overline{\mathcal{A}_2})$	2. construct TS $\mathcal{T}'$ with $Traces_{fin}(\mathcal{T}') = \ldots$
3. check if $\mathcal{L}(\mathcal{A}) = \varnothing$	3. invariant checking for $\mathcal{T}'$

Checking regular safety properties



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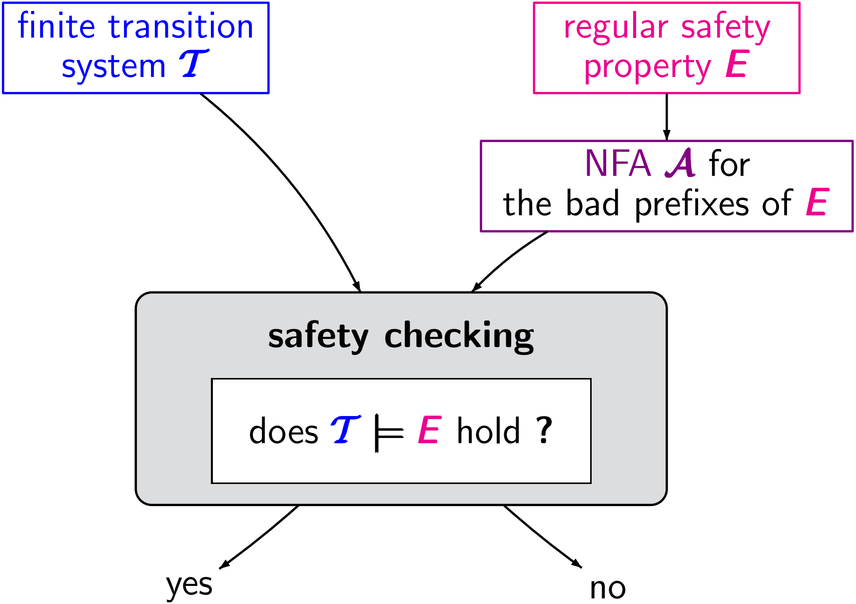
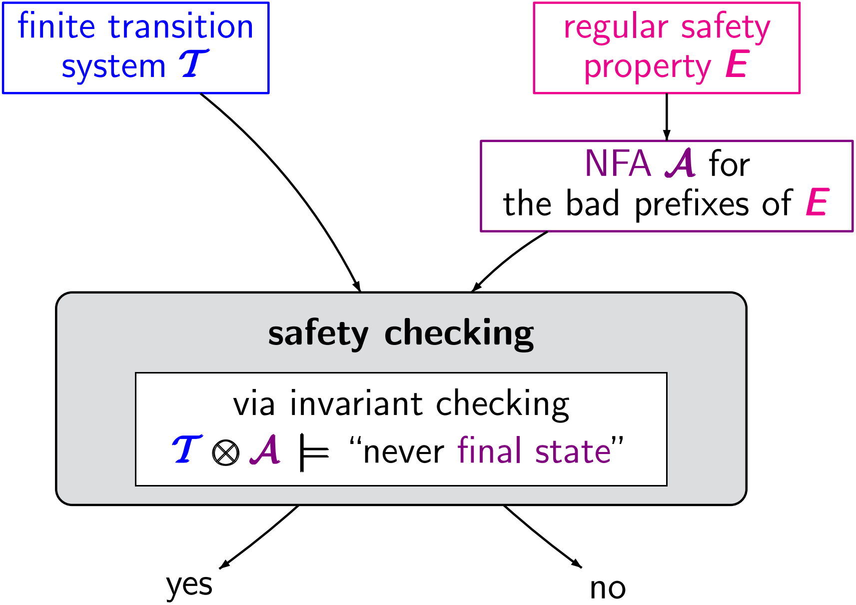
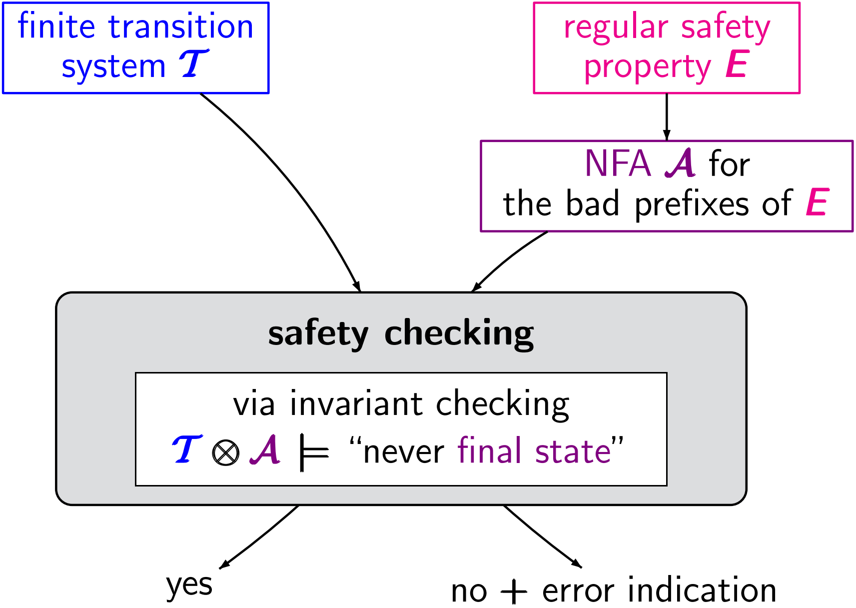


Checking regular safety properties



Checking regular safety properties



Product of a TS and an NFA

Product of a TS and an NFA

finite transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

path
fragment $\widehat{\pi}$

finite transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

s_0	$L(s_0) = A_0$
$\downarrow$	
s_1	$L(s_1) = A_1$
$\downarrow$	
s_2	$L(s_2) = A_2$
$\downarrow$	
$\vdots$	$\vdots$
$\downarrow$	
s_n	$L(s_n) = A_n$
path fragment $\widehat{\pi}$	trace

Product of a TS and an NFA

finite transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

path fragment $\widehat{\pi}$	trace	run for $trace(\widehat{\pi})$
s_0	$L(s_0) = A_0$	$q_0 \in Q_0$
$\downarrow$		$\downarrow A_0$
s_1	$L(s_1) = A_1$	q_1
$\downarrow$		$\downarrow A_1$
s_2	$L(s_2) = A_2$	q_2
$\downarrow$		$\downarrow A_2$
$\vdots$	$\vdots$	$\vdots$
$\downarrow$		$\downarrow$
s_n	$L(s_n) = A_n$	q_n
		$\downarrow A_n$
		q_{n+1}

Product of a TS and an NFA

finite transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes
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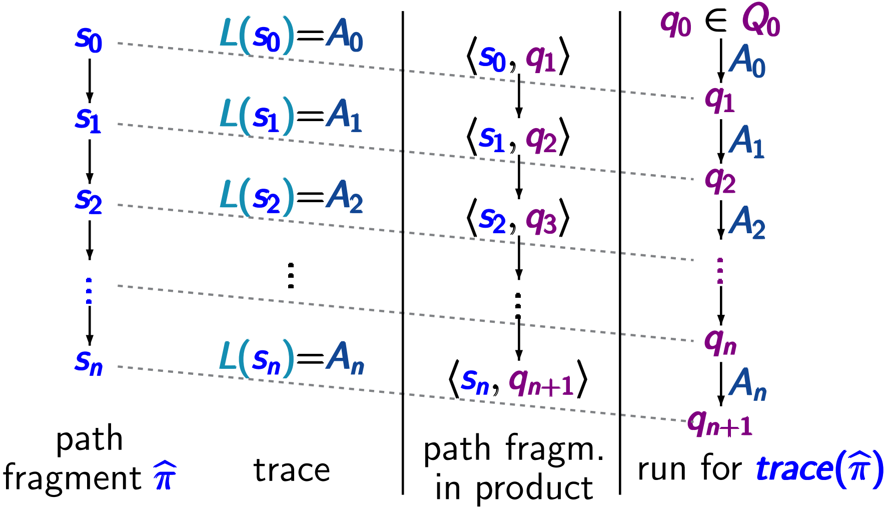

Product transition system

Product transition system

$$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$$ transition system

$$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$$ NFA

Product transition system

$$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L) \quad \text{transition system}$$

$$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F) \quad \text{NFA}$$

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S_0', AP', L')$

Product transition system

$$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L) \quad \text{transition system}$$

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$$\frac{s \xrightarrow{\alpha} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}{}' \langle s', q' \rangle}$$

Product transition system

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initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta\big(Q_0, L(s_0)\big) \}$

Product transition system

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initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta\big(Q_0, L(s_0)\big) \}$

for $P \subseteq Q$ and $A \subseteq AP$: $\delta(P, A) = \bigcup_{p \in P} \delta(p, A)$

Product transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S_0', AP', L')$

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set of atomic propositions: $AP' = Q$

Product transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S_0', AP', L')$

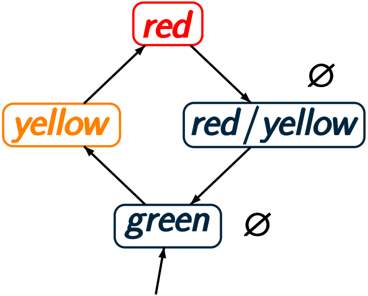
$$\frac{s \stackrel{\alpha}{\longrightarrow} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \stackrel{\alpha}{\longrightarrow}' \langle s', q' \rangle}$$

initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta\big(Q_0, L(s_0)\big) \}$

set of atomic propositions: $AP' = Q$

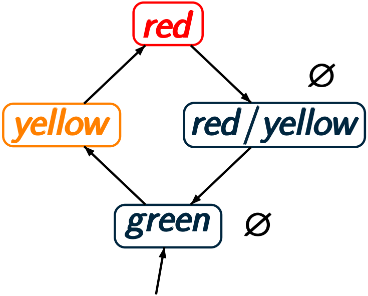
labeling function: $L'(\langle s, q \rangle) = \{q\}$

Example: product-TS



transition system $\mathcal{T}$ over
$AP = \{red, yellow\}$

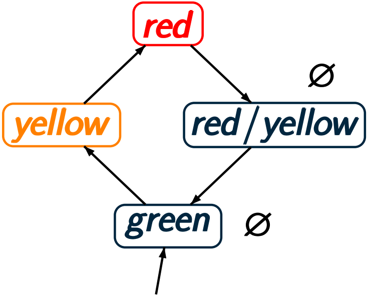
Example: product-TS

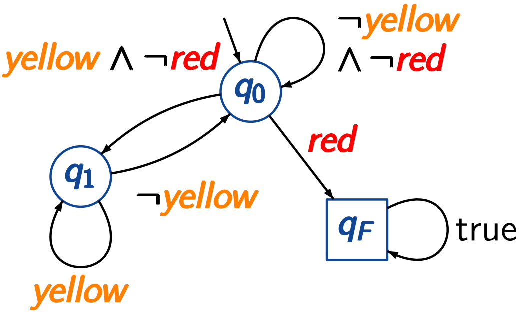


transition system $\mathcal{T}$ over
$AP = \{red, yellow\}$

> $\mathcal{T}$ satisfies the safety property E
> *"every red phase is preceded by a yellow phase"*

Example: product-TS



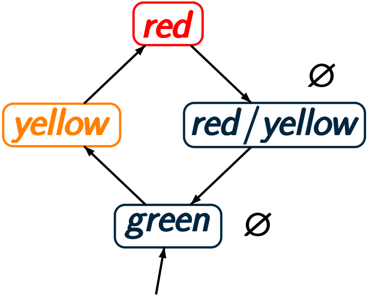


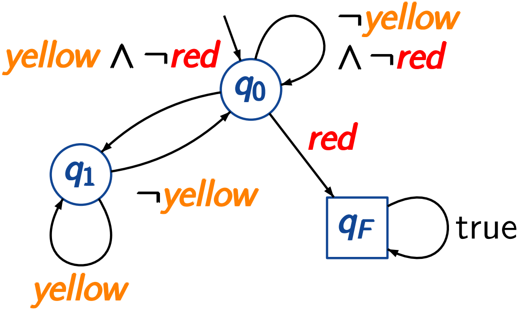
transition system $\mathcal{T}$ over $AP = \{red, yellow\}$

DFA $\mathcal{A}$ for the bad prefixes for E

$\mathcal{T}$ satisfies the safety property E
"every red phase is preceded by a yellow phase"

Example: product-TS





green q_0 red / yellow q_0

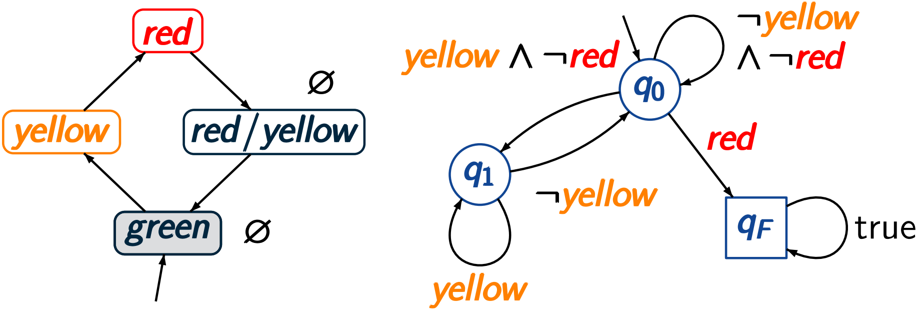
yellow q_1 red q_0

...

product-TS
$\mathcal{T} \otimes \mathcal{A}$

$(4 * 3 = 12$ states$)$

Example: product-TS



green q_0	red / yellow q_0
yellow q_1	red q_0

...

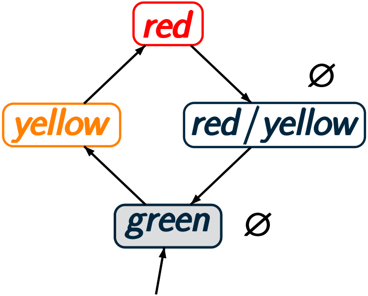
initial state

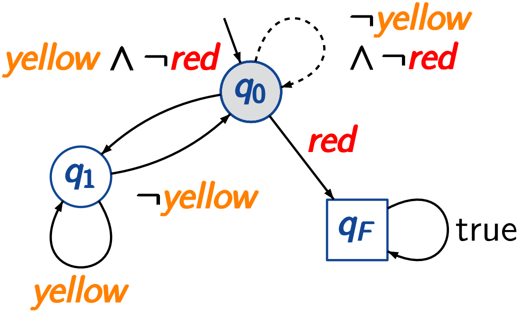
$\langle green, \delta(q_0, \varnothing)\rangle$

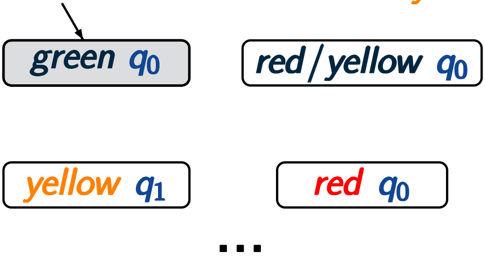
↑

$L(green) = \varnothing$

Example: product-TS



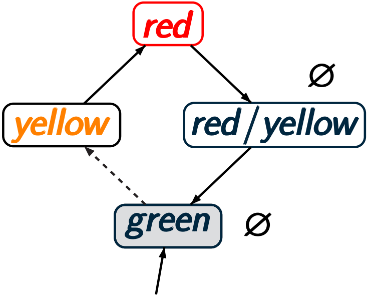


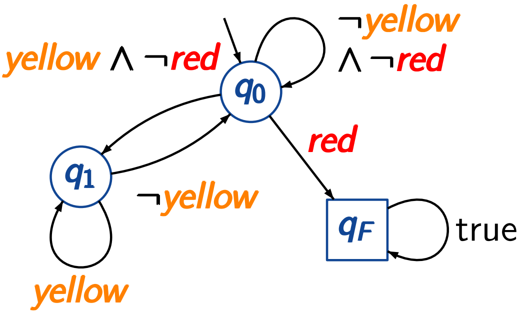


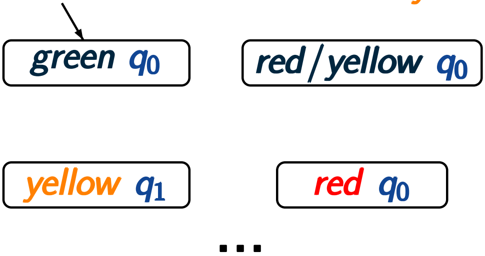
initial state

$$\langle green, \underbrace{\delta(q_0, \varnothing)}_{= q_0} \rangle$$

Example: product-TS

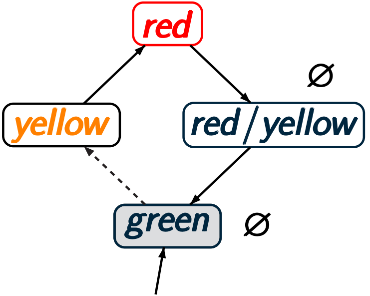


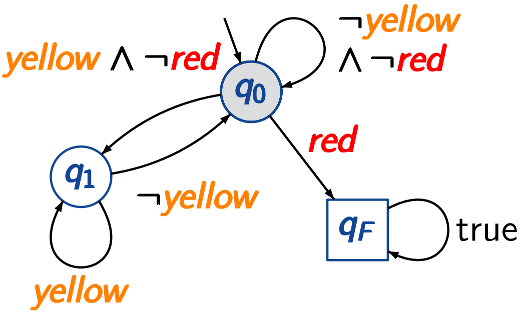


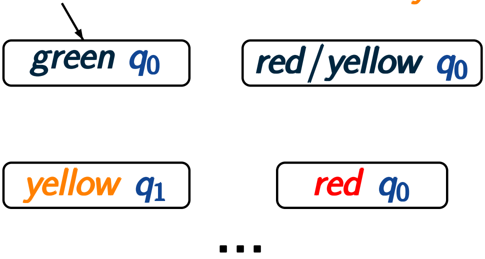


lifting the transition
green ⟶ yellow

Example: product-TS







lifting the transition

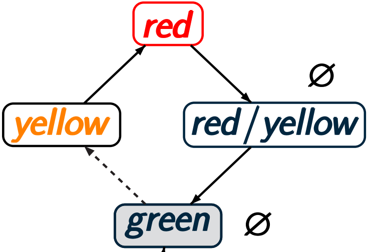
green $\longrightarrow$ *yellow*

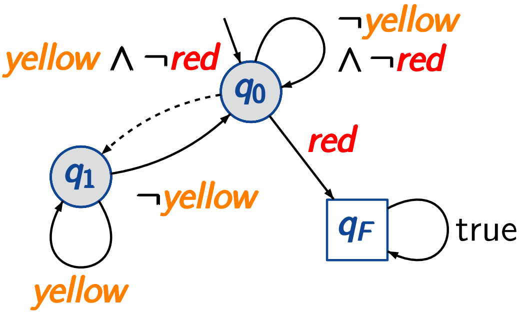
$\langle$ *green*, q_0 $\rangle$

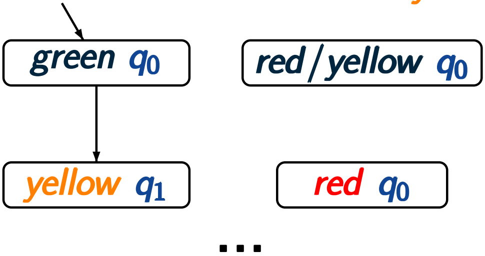
$\downarrow$

$\langle$ *yellow*, ? $\rangle$

Example: product-TS







lifting the transition

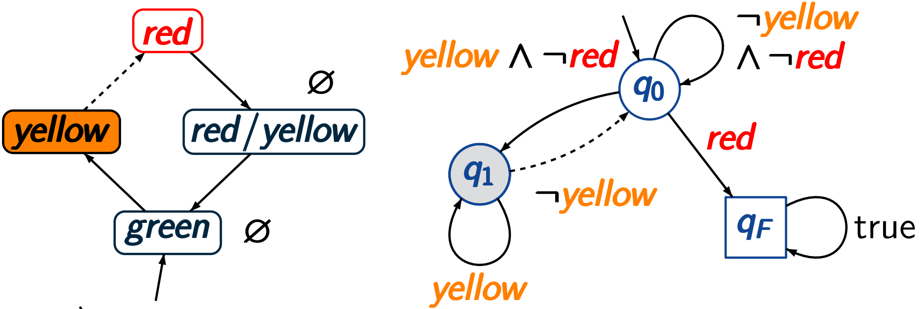
green $\longrightarrow$ yellow

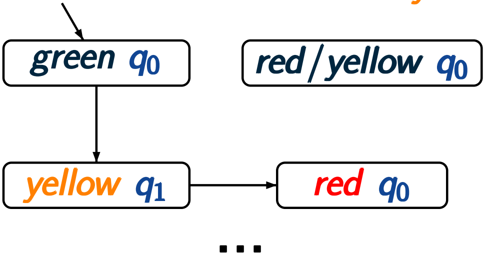
$$\langle green, q_0 \rangle$$

$$\downarrow$$

$$\langle yellow, \underbrace{\delta(q_0, \{yellow\})}_{= q_1} \rangle$$

Example: product-TS





lifting the transition

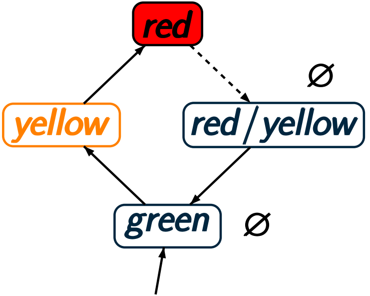
$$yellow \longrightarrow red$$

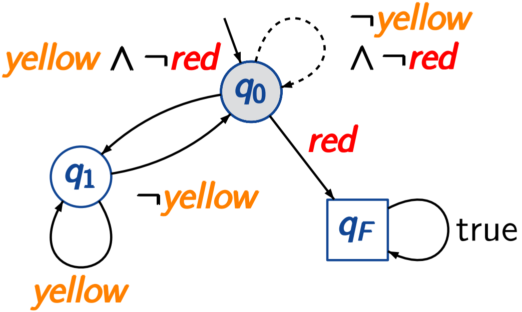
$$\langle yellow, q_1 \rangle$$

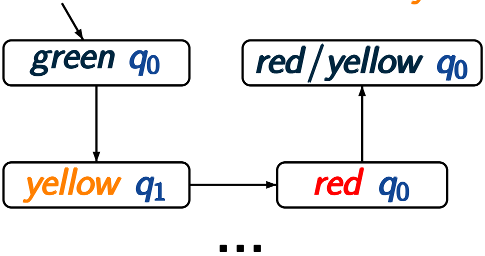
$$\downarrow$$

$$\langle red, \underbrace{\delta(q_1, \{red\})}_{= q_0} \rangle$$

Example: product-TS







lifting the transition

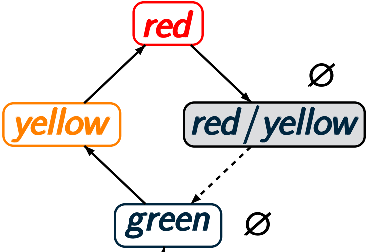
red $\longrightarrow$ *red / yellow*

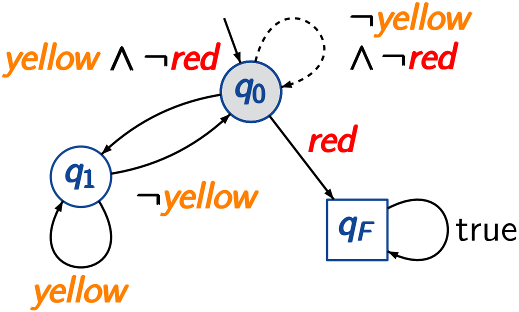
$\langle$*red*, $q_0\rangle$

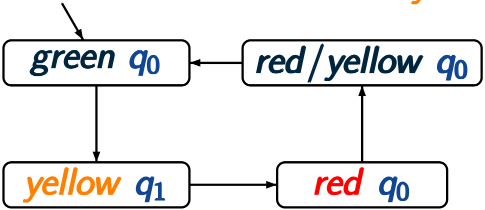
$\Downarrow$

$\langle$*red / yellow*, $\underbrace{\delta(q_0, \varnothing)}_{= q_0}\rangle$

Example: product-TS





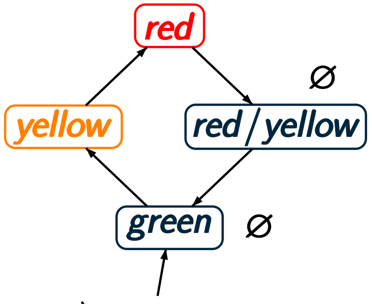


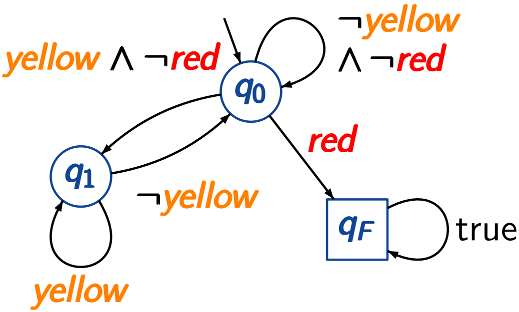
lifting the transition

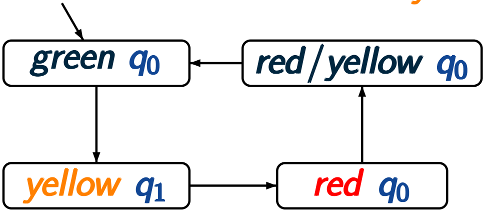
red / yellow $\longrightarrow$ *green*

$$\langle \mathit{red/yellow}, q_0 \rangle$$
$$\downarrow$$
$$\langle \mathit{green}, \underbrace{\delta(q_0, \varnothing)}_{=\, q_0} \rangle$$

Example: product-TS



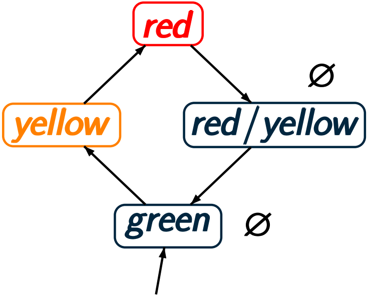


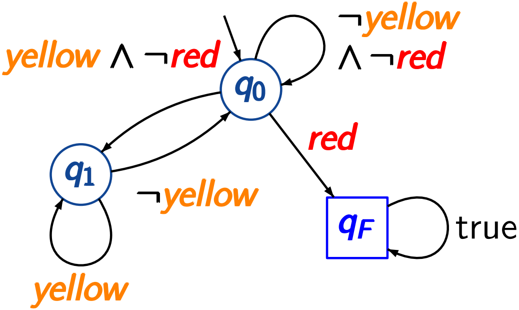


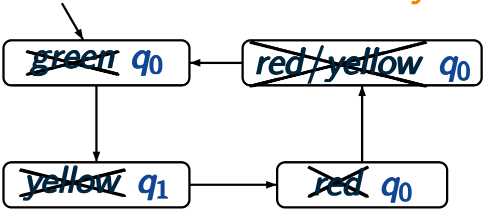
product-TS
$\mathcal{T} \otimes \mathcal{A}$

$\mathbf{4} * \mathbf{3} = \mathbf{12}$ states, but just **4** reachable states

Example: product-TS

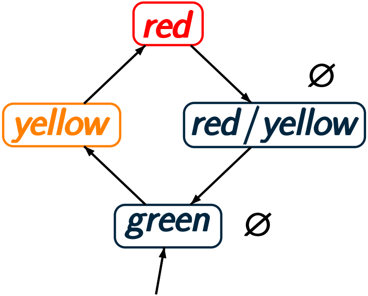


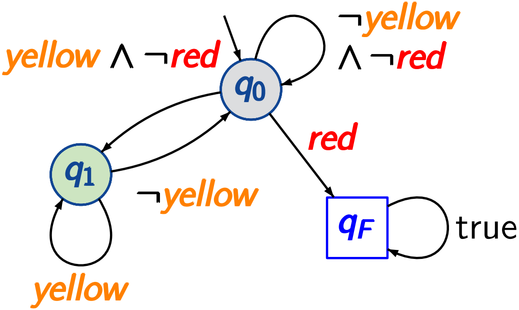


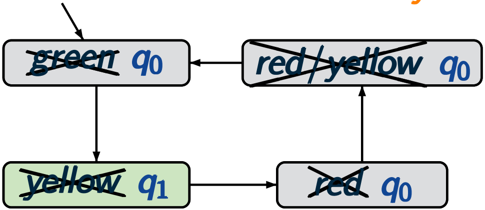


set of propositions
$AP' = \{q_0, q_1, q_F\}$

Example: product-TS







set of propositions
$AP' = \{q_0, q_1, q_F\}$

invariant condition $\neg q_F$ holds
for all reachable states

Technical remark on the product-TS

definition of the product of

- a transition system $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
- an NFA $\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

then the product $\mathcal{T} \otimes \mathcal{A} = (S \times Q, Act, \rightarrow', \ldots)$ is a TS

Technical remark on the product-TS

definition of the product of

- a transition system $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

  without terminal states

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Technical remark on the product-TS

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↑ without terminal states

Technical remark on the product-TS

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without terminal states

assumptions on the NFA $\mathcal{A}$:

Technical remark on the product-TS

definition of the product of

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  without terminal states

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without terminal states

assumptions on the NFA $\mathcal{A}$:

- $\mathcal{A}$ is non-blocking, i.e.,

$$Q_0 \neq \varnothing \;\wedge\; \forall q \in Q \, \forall A \in 2^{AP}. \, \delta(q, A) \neq \varnothing$$

Technical remark on the product-TS

definition of the product of

- a transition system $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

  without terminal states

- an NFA $\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

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without terminal states

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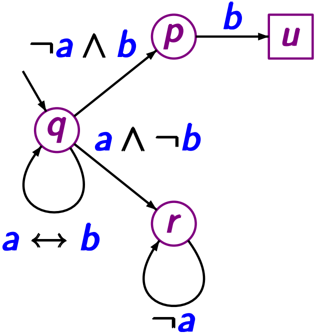
- $\mathcal{A}$ is non-blocking, i.e.,

  $$Q_0 \neq \varnothing \ \wedge \ \forall q \in Q \, \forall A \in 2^{AP}. \, \delta(q, A) \neq \varnothing$$

- no initial state of $\mathcal{A}$ is final, i.e., $Q_0 \cap F = \varnothing$

Non-blocking NFA

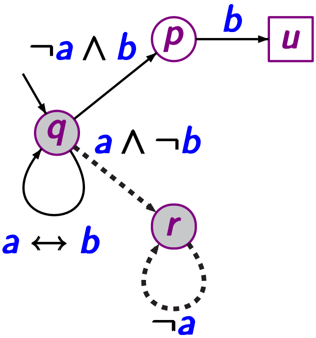
NFA $\mathcal{A}$



alphabet $\Sigma = 2^{AP}$ where $AP = \{a, b\}$

Non-blocking NFA

NFA $\mathcal{A}$

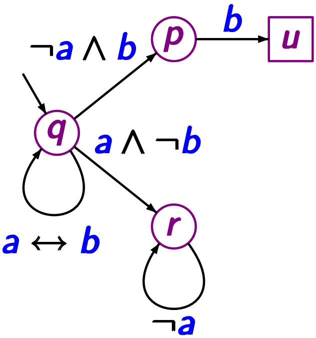


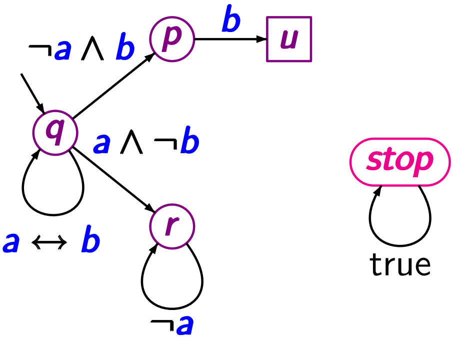
blocks for input
$\{a\}\,\varnothing\,\{a\}$

alphabet $\Sigma = 2^{AP}$ where $AP = \{a, b\}$

Non-blocking NFA

NFA $\mathcal{A}$ $\rightsquigarrow$ equivalent NFA $\mathcal{A}'$





blocks for input $\{a\}\varnothing\{a\}$

add a trap state *stop*

Non-blocking NFA

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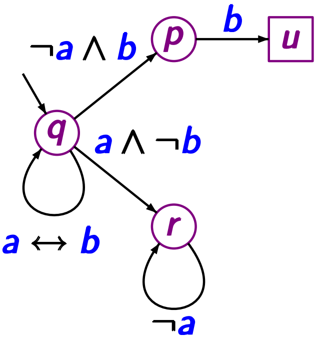

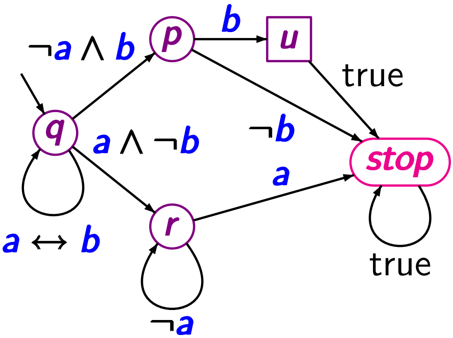

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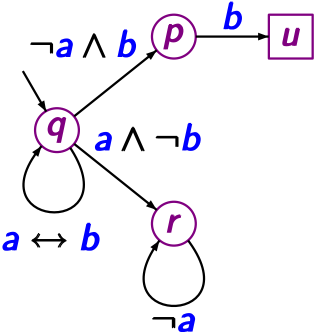

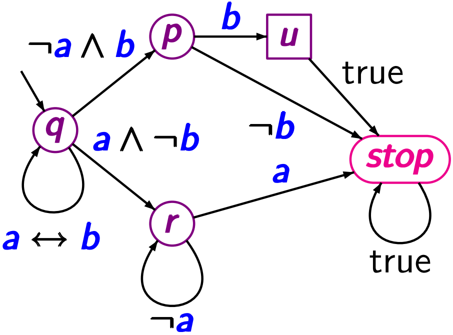

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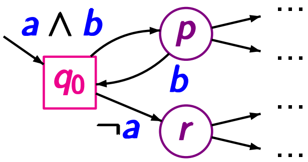
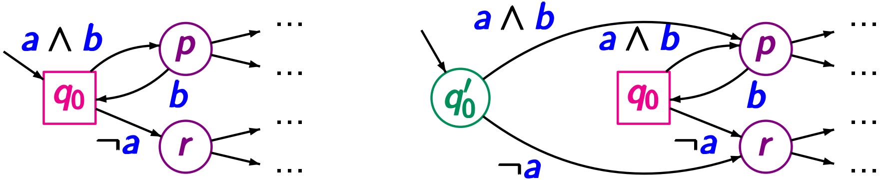
NFA where no initial state is final

NFA $\mathcal{A}$ with $Q_0 \cap F \neq \varnothing$



NFA where no initial state is final

NFA $\mathcal{A}$ with $Q_0 \cap F \neq \varnothing$ $\rightsquigarrow$ NFA $\mathcal{A}'$ with $Q_0 \cap F = \varnothing$



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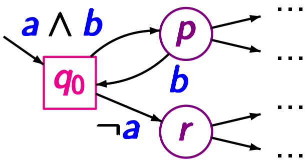

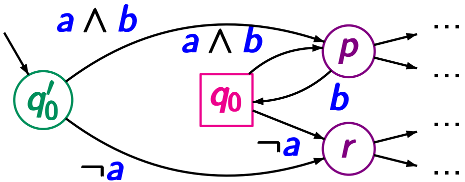

$$\mathcal{L}(\mathcal{A}') = \mathcal{L}(\mathcal{A}) \setminus \{\varepsilon\}$$

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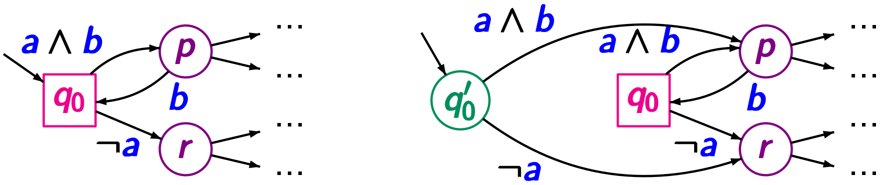

$$\mathcal{L}(\mathcal{A}') = \mathcal{L}(\mathcal{A}) \setminus \{\varepsilon\}$$

note: if $\mathcal{A}$ is an NFA for the bad prefixes of a safety property then

$$\varepsilon \notin \mathcal{L}(\mathcal{A}) = \mathit{BadPref}$$

Model checking regular safety properties

IS2.5-26A

Model checking regular safety properties

… via a reduction to invariant checking …..

Model checking regular safety properties

Let $\mathcal{T} = (S, Act, \to, S_0, AP, L)$ be a transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ be an NFA
for the bad prefixes of a regular safety property E

Model checking regular safety properties

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where "$\neg F$" denotes $\bigwedge_{q \in F} \neg q$

Product transition system

$\mathcal{T} = (S, Act, \to, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S_0', AP', L')$

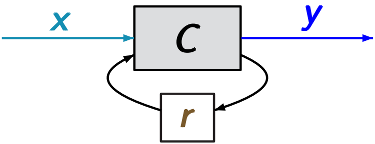
$$\frac{s \xrightarrow{\alpha} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}{}' \langle s', q' \rangle}$$

initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta\big(Q_0, L(s_0)\big) \}$

set of atomic propositions: $AP' = Q$

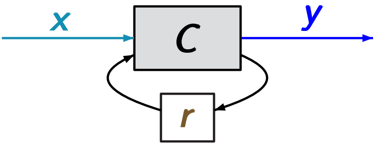
labeling function: $L'(\langle s, q \rangle) = \{q\}$

Example: sequential circuit



$$\lambda_y = \delta_r = x \oplus r$$

Example: sequential circuit



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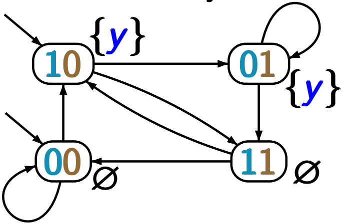
initially $r = 0$

transition system $\mathcal{T}$



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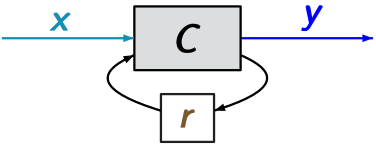

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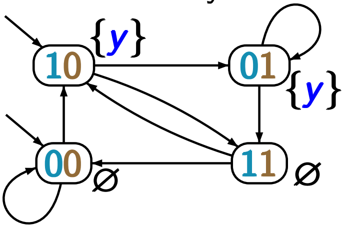

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safety property E

The circuit will never ouput two ones after each other

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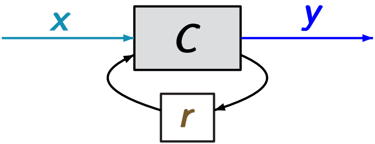

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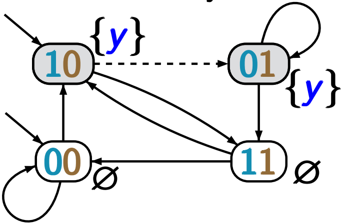

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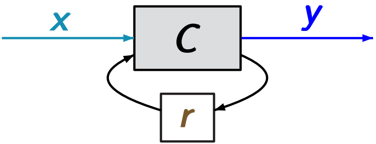

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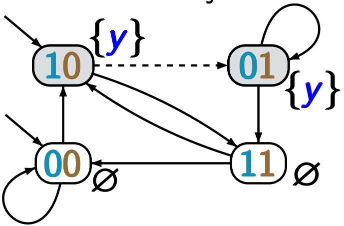

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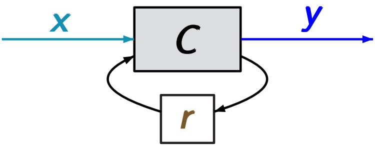

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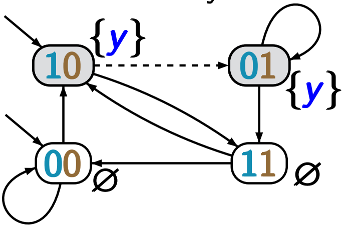

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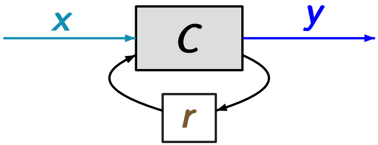

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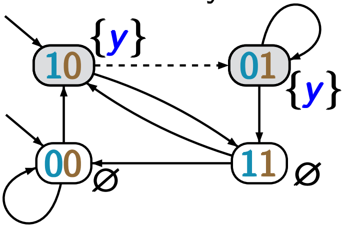

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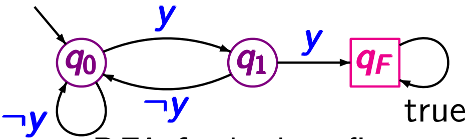

DFA for bad prefixes

safety property E

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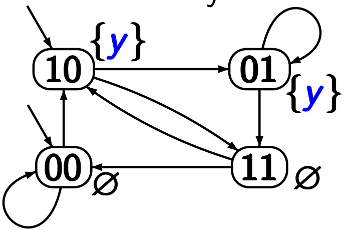
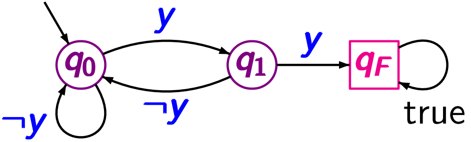
Example: product-TS

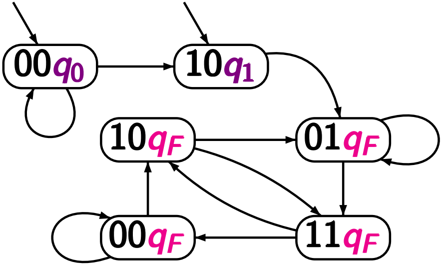
transition system $\mathcal{T}$



safety property E

… *never two ones in a row* …





product-TS $\mathcal{T} \otimes \mathcal{A}$

Example: product-TS

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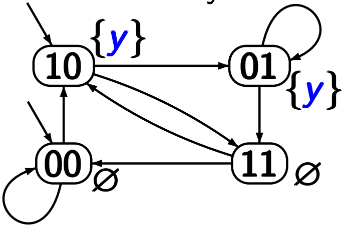

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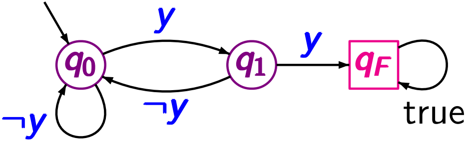

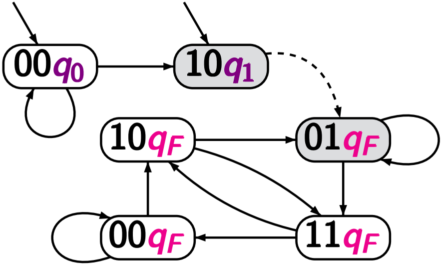

$\mathcal{T} \otimes \mathcal{A} \not\models$ "never q_F"

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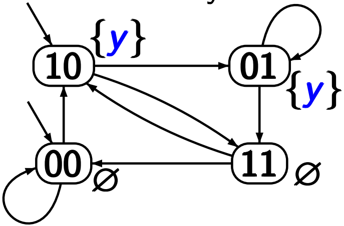

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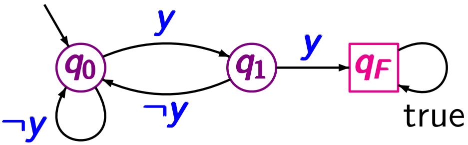

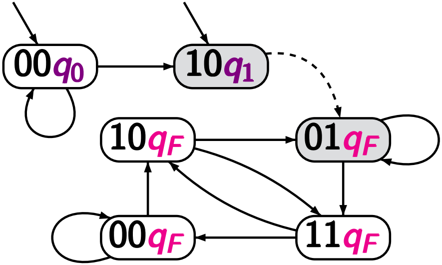

error indication for

$\mathcal{T} \otimes \mathcal{A} \not\models$ "never q_F"

$10q_1 \quad 01q_F$

Example: product-TS

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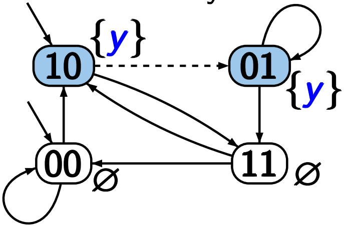

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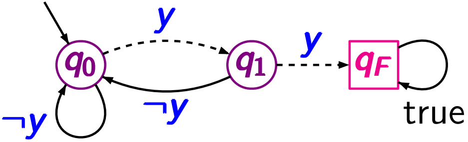

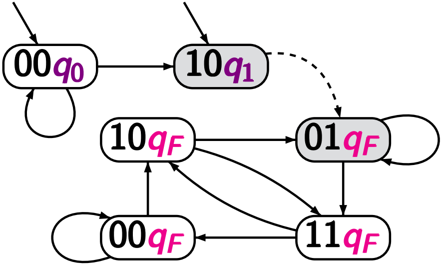

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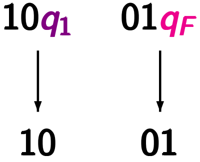

Model checking regular safety properties

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input: finite TS $\mathcal{T}$,
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output: "yes" if $\mathcal{T} \models E$
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check whether $\mathcal{T} \otimes \mathcal{A} \models$ "always $\neg F$"

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```
IF T ⊗ A ⊨ "always ¬F"
 THEN  return "yes"
 ELSE



FI
```

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IF $\mathcal{T} \otimes \mathcal{A} \models$ “always $\neg F$”

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$\langle s_0, p_0 \rangle \langle s_1, p_1 \rangle \ldots \langle s_n, p_n \rangle$ where $p_n \in F$

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FI

time complexity: $\mathcal{O}\big(size(\mathcal{T}) \cdot size(\mathcal{A})\big)$

Correct or wrong?

If $\mathcal{T}$ is a finite transition system then $\mathit{Traces}_{\mathit{fin}}(\mathcal{T})$ is regular.

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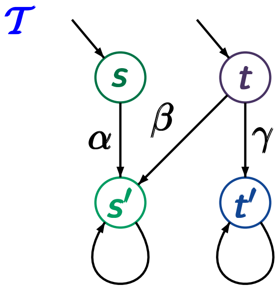

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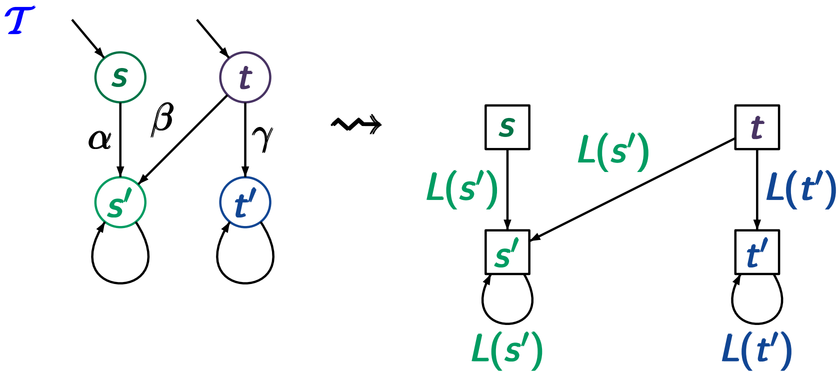

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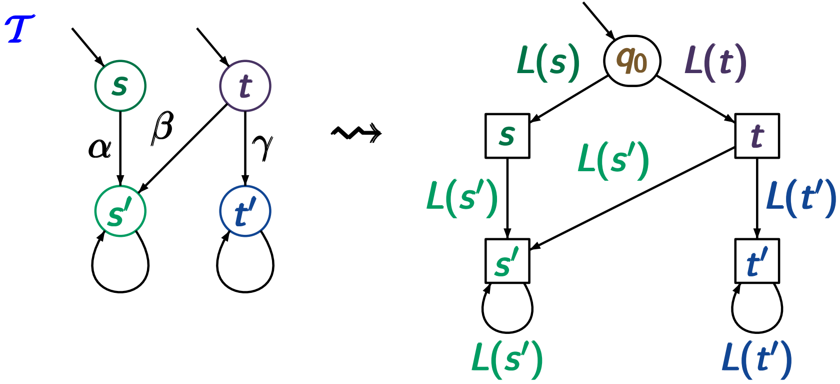

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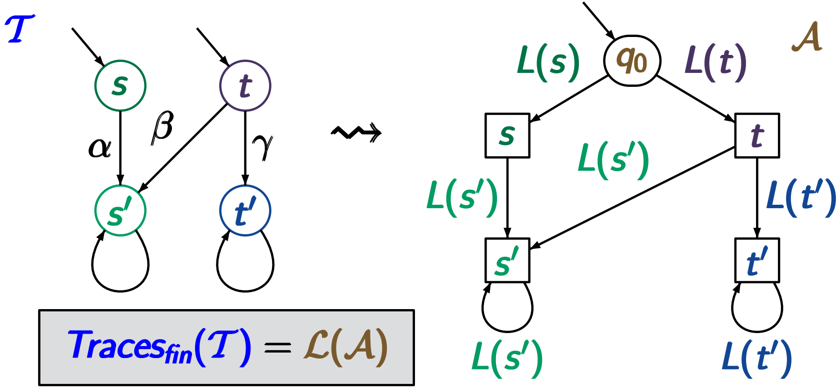

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