Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

- regular safety properties
- ω-regular properties
- model checking with Büchi automata $\longleftarrow$

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

Verifying ω-regular properties

given: finite transition system $\mathcal{T}$
ω-regular property E

question: does $\mathcal{T} \models E$ hold ?

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requires techniques for checking **persistence properties** in finite TS

Persistence property

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Let E be an LT-property, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$

E is called a persistence property if there exists a propositional formula Φ over AP such that

$$E = \left\{ \begin{array}{c} \text{set of all infinite words } A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega} \\ \text{s.t.} \quad \overset{\infty}{\forall}\, i \geq 0.\ A_i \models \Phi \end{array} \right.$$

$\overset{\infty}{\forall}\, i{\geq}0. \ldots = \exists j{\geq}0\ \forall i{\geq}j. \ldots$ "for all but finitely many"

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"from some moment on Φ"
"eventually forever Φ"

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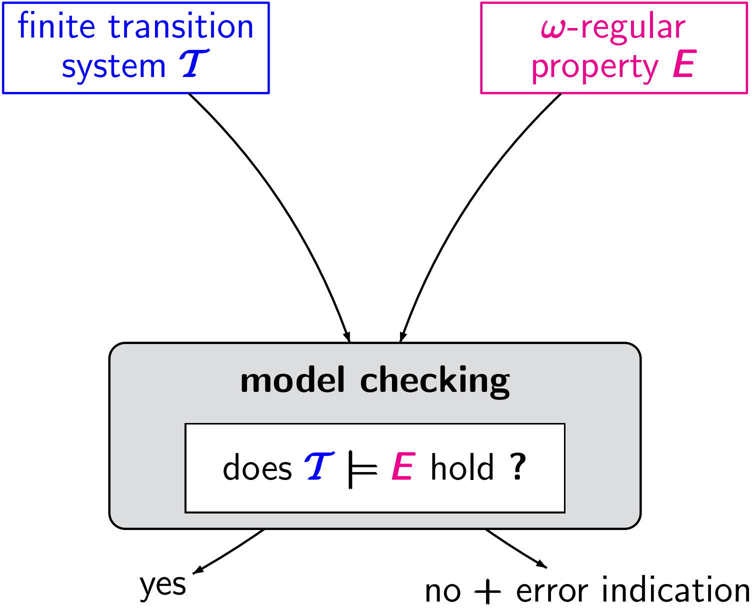
Checking ω-regular properties



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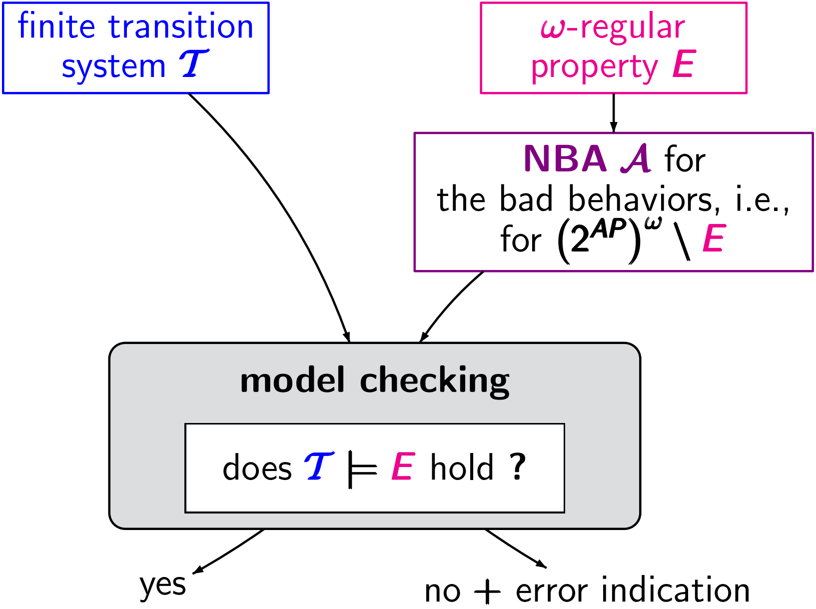

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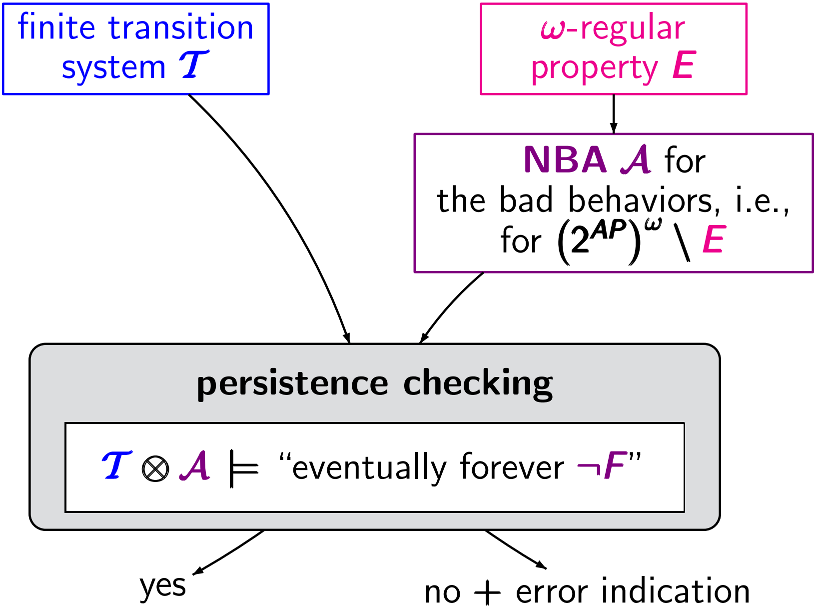

Recall: product of a TS and an NFA

finite transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

path
fragment $\widehat{\pi}$

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s_0	$L(s_0) = A_0$
$\downarrow$	
s_1	$L(s_1) = A_1$
$\downarrow$	
s_2	$L(s_2) = A_2$
$\downarrow$	
$\vdots$	$\vdots$
$\downarrow$	
s_n	$L(s_n) = A_n$
path fragment $\widehat{\pi}$	trace

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$s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots \rightarrow s_n$

$L(s_0) = A_0$
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$\vdots$
$L(s_n) = A_n$

$q_0 \in Q_0 \xrightarrow{A_0} q_1 \xrightarrow{A_1} q_2 \xrightarrow{A_2} \ldots \rightarrow q_n \xrightarrow{A_n} q_{n+1}$

path fragment $\widehat{\pi}$ | trace | run for $trace(\widehat{\pi})$

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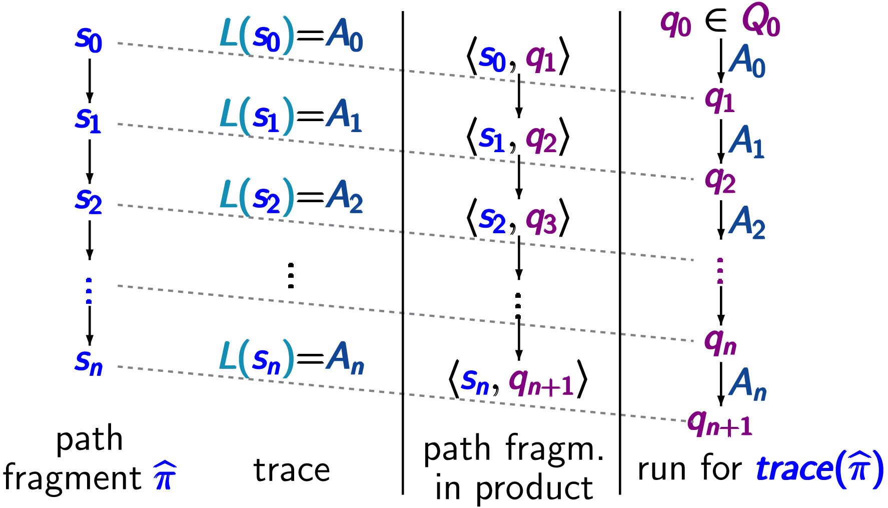

Product transition system

recall: definition of the product of a **TS** and **NFA**

Product transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S'_0, AP', L')$

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initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta\big(Q_0, L(s_0)\big) \}$

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set of atomic propositions: $AP' = Q$

labeling function: $L'(\langle s, q \rangle) = \{q\}$

Product transition system

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$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA $\longleftarrow$ same definition for **NBA**

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Product of a TS and NBA

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$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ **NFA** or **NBA**

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relies on a reduction to the persistence checking problem

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ finite transition system without terminal states

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ non-blocking NBA

representing the bad behaviors of an ω-regular LT-property E

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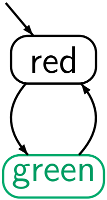
Example: ω-regular model checking

TS $\mathcal{T}$



LT property: "infinitely often green"

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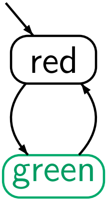

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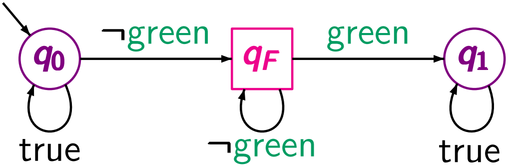
NBA $\mathcal{A}$ for the complement

"from some moment on ¬green"



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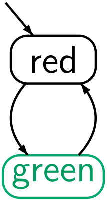

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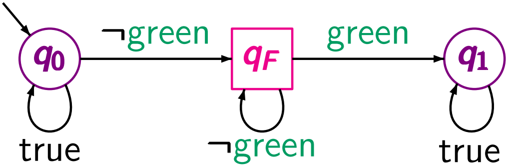

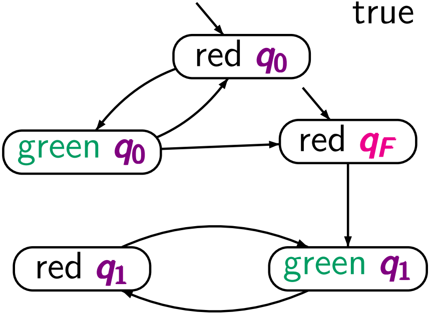

reachable fragment of the product TS $\mathcal{T} \otimes \mathcal{A}$

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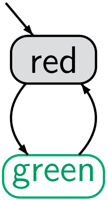

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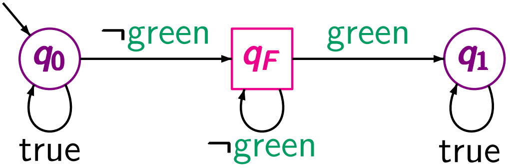

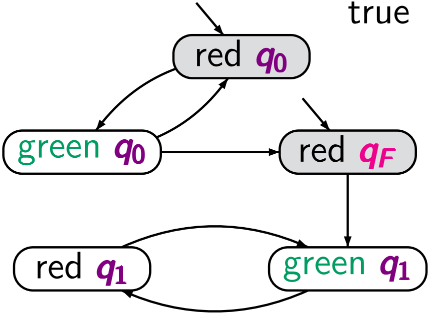

initial states:

$\langle \text{red}, q \rangle$ where

$$q \in \delta(q_0, L(\text{red})) = \delta(q_0, \varnothing) = \{q_0, q_F\}$$

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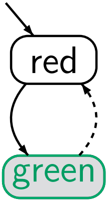

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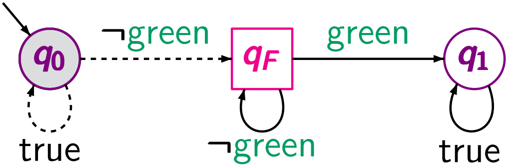

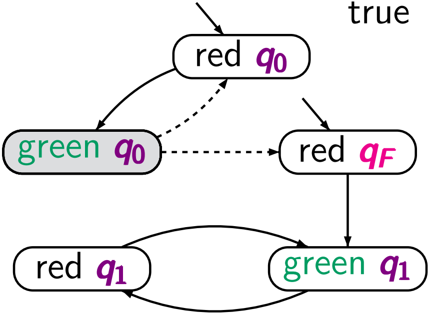

transition

$\langle \text{green}, q_0 \rangle \longrightarrow \langle \text{red}, q \rangle$

$$q \in \delta(q_0, L(\text{red}))$$
$$= \delta(q_0, \varnothing)$$
$$= \{q_0, q_F\}$$

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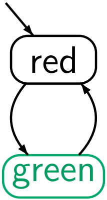

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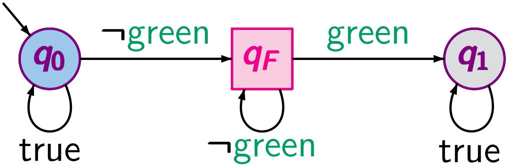

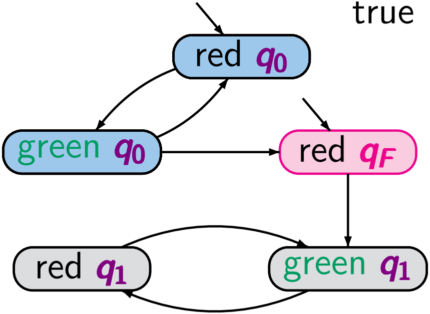

atomic propositions

$AP' = \{q_0, q_F, q_1\}$

obvious labeling function

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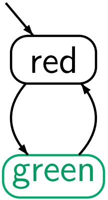

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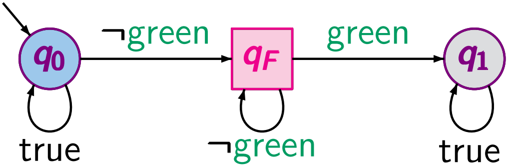

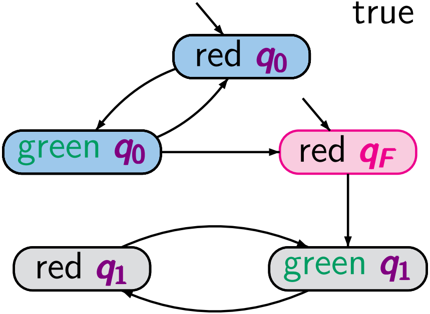

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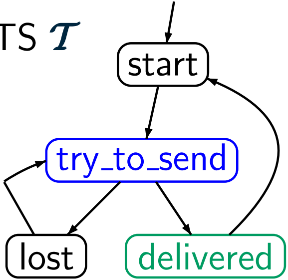

ω-regular LT property E:

"each (repeatedly) sent message will eventually be delivered"

Example: ω-regular model checking

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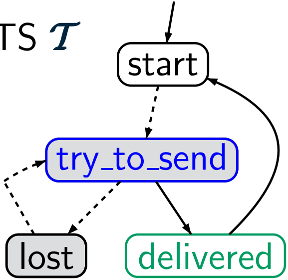

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$\mathcal{T} \not\models E$

Example: ω-regular model checking

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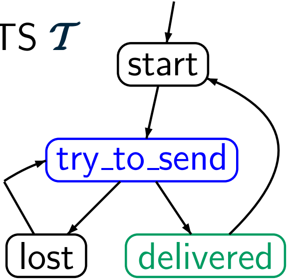
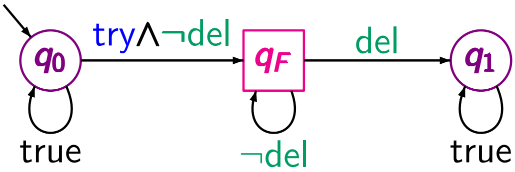


NBA $\mathcal{A}$ for the bad behaviors



ω-regular LT property E:

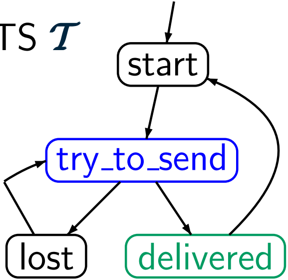
"each (repeatedly) sent message will eventually be delivered"

complement of E, i.e., LT property for the bad behaviors:

"never delivered after some trial"

Example: ω-regular model checking

TS $\mathcal{T}$



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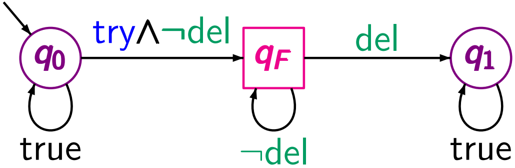

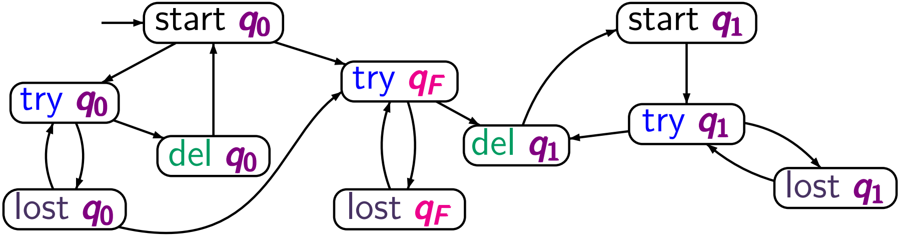

reachable fragment of the product-TS

Example: ω-regular model checking

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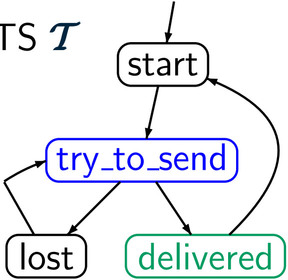

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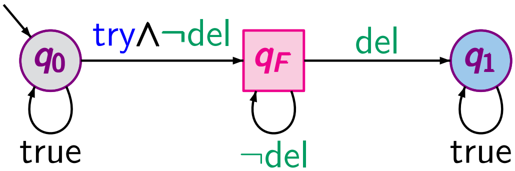

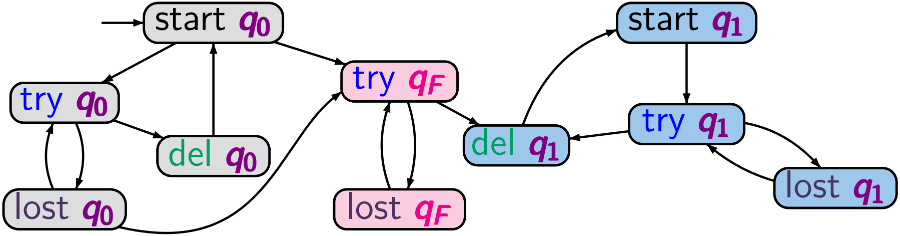

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Example: ω-regular model checking

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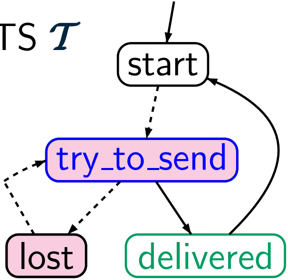

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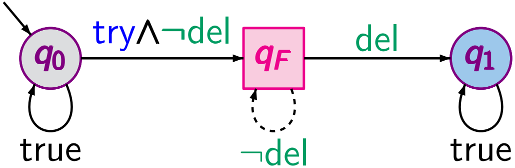

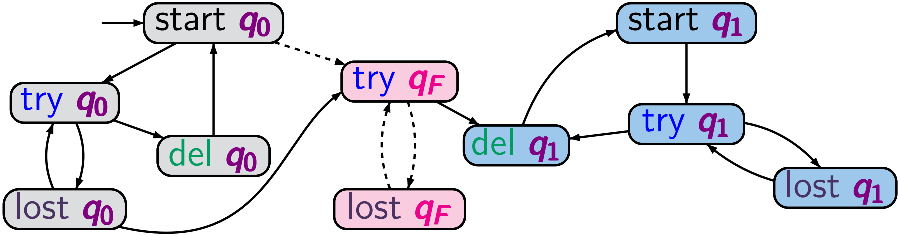

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for regular safety property E

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iff $Traces_{fin}(\mathcal{T}) \cap BadPref = \varnothing$

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SCC: strongly connected component, i.e., maximal set of states that are reachable from each other

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A SCC is called non-trivial if it has at least one edge.
"either 1 state with a self-loop or 2 or more states"

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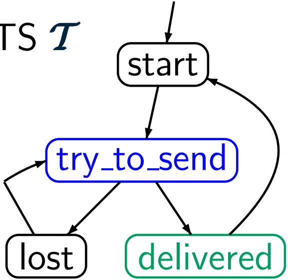
method: calculate and analyze the **SCCs**

Example: ω-regular model checking

TS $\mathcal{T}$



ω-regular LT property E:

"each (repeatedly) sent message will eventually be delivered"

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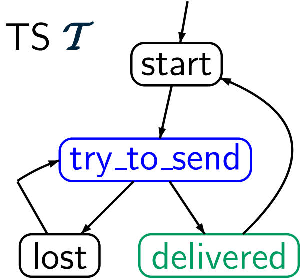

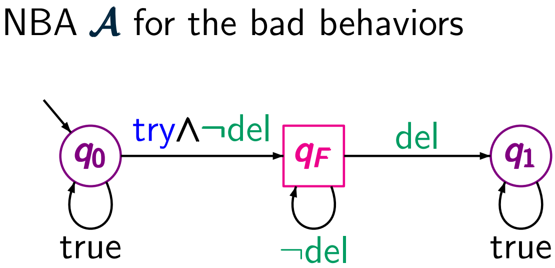

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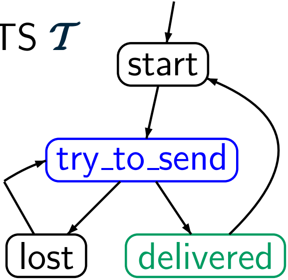
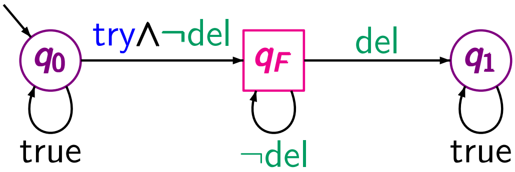


NBA $\mathcal{A}$ for the bad behaviors



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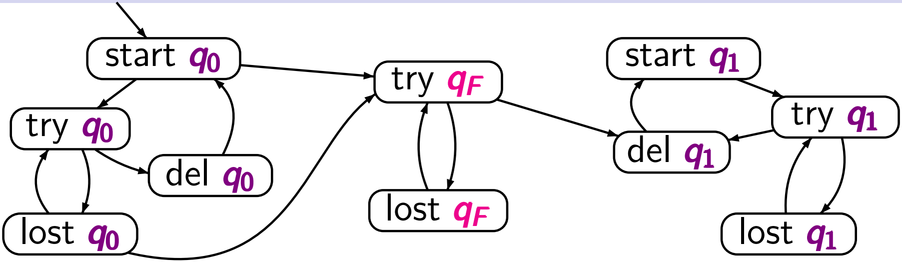
... analysis of the **SCCs** in product $\mathcal{T} \otimes \mathcal{A}$...

Example: persistence checking $\mathcal{T} \otimes \mathcal{A}$



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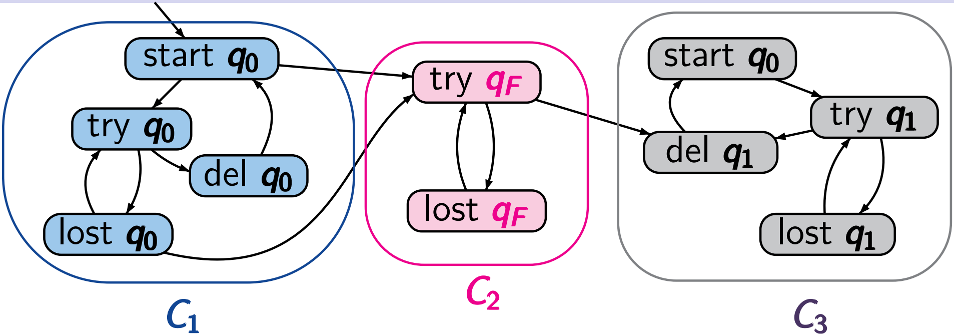

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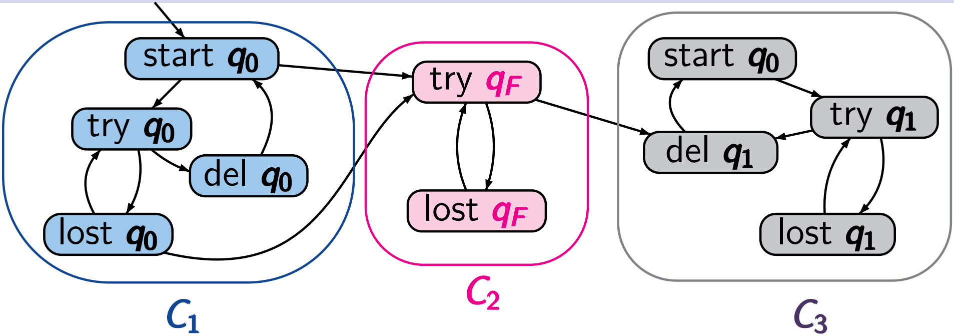

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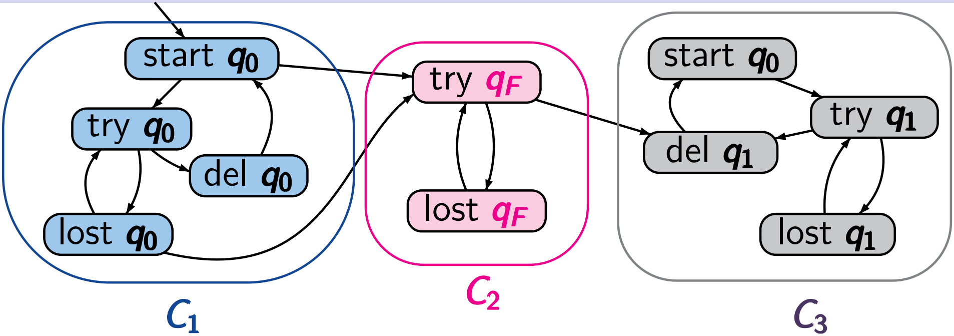

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Persistence checking algorithms based on ...

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