Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL ⟵
- automata-based LTL model checking
- complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

Linear Temporal Logic (LTL)

Linear Temporal Logic (LTL)

$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc \varphi \ \big| \ \varphi_1 \, \mathsf{U} \, \varphi_2$$

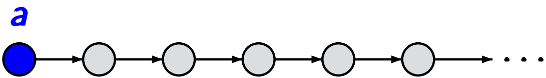
where $a \in AP$ $\qquad \bigcirc \widehat{=}$ next $\qquad \mathsf{U} \widehat{=}$ until

Linear Temporal Logic (LTL)

$$\varphi \ ::= \ \mathit{true} \ \mid \ a \ \mid \ \varphi_1 \wedge \varphi_2 \ \mid \ \neg\varphi \ \mid \ \bigcirc \varphi \ \mid \ \varphi_1 \, \mathsf{U} \, \varphi_2$$

where $a \in AP$ $\quad\quad$ $\bigcirc \mathrel{\widehat{=}}$ next $\quad\quad$ $\mathsf{U} \mathrel{\widehat{=}}$ until

atomic
proposition
$a \in AP$

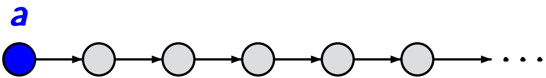


Linear Temporal Logic (LTL)

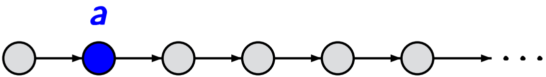
$$\varphi \;::=\; \mathit{true} \;\big|\; a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \neg\varphi \;\big|\; \bigcirc\varphi \;\big|\; \varphi_1 \,\mathsf{U}\, \varphi_2$$

where $a \in AP$ $\qquad \bigcirc \;\widehat{=}\;$ next $\qquad \mathsf{U} \;\widehat{=}\;$ until

atomic proposition $a \in AP$



next operator $\bigcirc a$



Linear Temporal Logic (LTL)

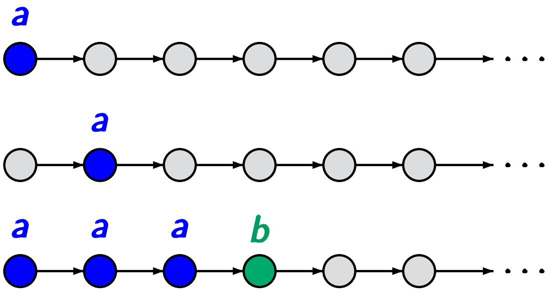
$$\varphi \ ::= \ \textit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc\varphi \ \big| \ \varphi_1 \, \mathsf{U} \, \varphi_2$$

where $a \in AP$ $\qquad \bigcirc \mathrel{\widehat{=}}$ next $\qquad \mathsf{U} \mathrel{\widehat{=}}$ until

atomic proposition $a \in AP$

next operator $\bigcirc a$

until operator $a \, \mathsf{U} \, b$



Derived operators in LTL

$$\varphi \ ::= \ \textit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc \varphi \ \big| \varphi_1 \, \mathsf{U} \, \varphi_2$$

derived operators:

$\vee, \rightarrow, \ldots$ as usual

Derived operators in LTL

$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc \varphi \ \big| \varphi_1 \, \mathsf{U} \, \varphi_2$$

derived operators:

$\vee, \rightarrow, \ldots$ as usual

$\Diamond\varphi \ \stackrel{\text{def}}{=} \ \mathit{true}\,\mathsf{U}\,\varphi$ eventually

Derived operators in LTL

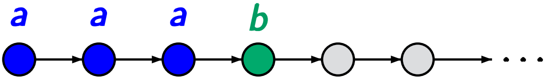
$$\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

derived operators:

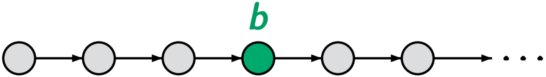
$\vee, \rightarrow, \ldots$ as usual

$$\Diamond\varphi \ \stackrel{\text{def}}{=} \ \mathit{true}\,\mathsf{U}\,\varphi \quad \text{eventually}$$

until operator
$a\,\mathsf{U}\,b$



eventually
$\Diamond b$



Derived operators in LTL

$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc\varphi \ \big| \ \varphi_1 \,\mathsf{U}\, \varphi_2$$

derived operators:

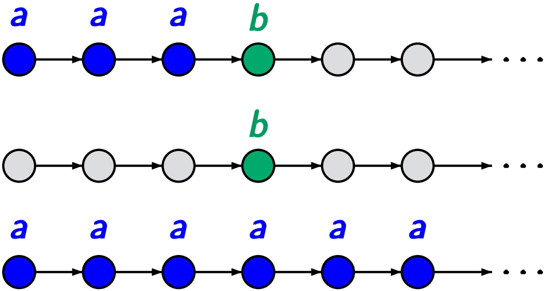
$\vee, \rightarrow, \ldots$ as usual

$\Diamond\varphi$	$\stackrel{\text{def}}{=}$	$\mathit{true}\,\mathsf{U}\,\varphi$	eventually
$\Box\varphi$	$\stackrel{\text{def}}{=}$	$\neg\Diamond\neg\varphi$	always

until operator $a\,\mathsf{U}\,b$

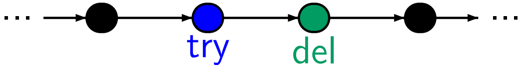
eventually $\Diamond b$

always $\Box a$



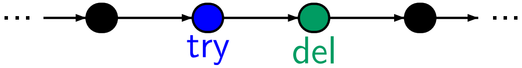
Next $\bigcirc$, until U and eventually $\Diamond$

$\Box$ (try_to_send $\rightarrow$ $\bigcirc$ delivered)

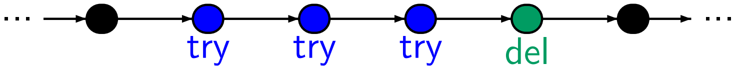


Next $\bigcirc$, until U and eventually $\Diamond$

$\Box\,(\mathit{try\_to\_send} \rightarrow \bigcirc\ \mathit{delivered})$

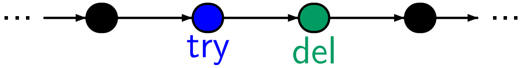


$\Box\,(\mathit{try\_to\_send} \rightarrow \mathit{try\_to\_send}\ \mathsf{U}\ \mathit{delivered})$

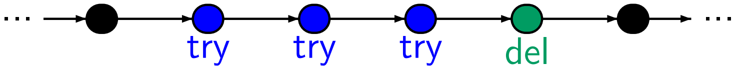


Next $\bigcirc$, until U and eventually $\Diamond$

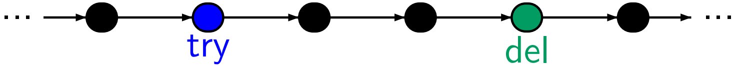
$$\Box\,(\text{try\_to\_send} \longrightarrow \bigcirc\ \text{delivered})$$



$$\Box\,(\text{try\_to\_send} \longrightarrow \text{try\_to\_send}\ \mathsf{U}\ \text{delivered})$$



$$\Box\,(\text{try\_to\_send} \longrightarrow \Diamond\ \text{delivered})$$



Examples for LTL formulas

$$\boxed{\varphi \;::=\; \mathit{true} \;\big|\; a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \neg\varphi \;\big|\; \bigcirc\varphi \;\big|\; \varphi_1 \,\mathsf{U}\, \varphi_2}$$

eventually

$$\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true}\,\mathsf{U}\,\varphi$$

always

$$\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas

$$\boxed{\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2}$$

eventually

$$\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true} \, \mathsf{U} \, \varphi$$

always

$$\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas:

mutual exclusion: $\Box(\neg \mathit{crit}_1 \vee \neg \mathit{crit}_2)$

Examples for LTL formulas

$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc \varphi \ \big| \ \varphi_1 \, \mathsf{U} \, \varphi_2$$

eventually

$$\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true} \, \mathsf{U} \, \varphi$$

always

$$\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas:

mutual exclusion: $\Box(\neg \mathit{crit}_1 \vee \neg \mathit{crit}_2)$

railroad-crossing: $\Box(\mathit{train\_is\_near} \rightarrow \mathit{gate\_is\_closed})$

Examples for LTL formulas

$$\varphi \quad ::= \quad \mathit{true} \;\big|\; a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \neg\varphi \;\big|\; \bigcirc\,\varphi \;\big|\; \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually

$$\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true}\,\mathsf{U}\,\varphi$$

always

$$\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas:

mutual exclusion: $\Box(\neg \mathit{crit}_1 \vee \neg \mathit{crit}_2)$

railroad-crossing: $\Box(\mathit{train\_is\_near} \rightarrow \mathit{gate\_is\_closed})$

progress property: $\Box(\mathit{request} \rightarrow \Diamond \mathit{response})$

Examples for LTL formulas

$$\boxed{\varphi \;::=\; \mathit{true} \;\big|\; a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \neg\varphi \;\big|\; \bigcirc\,\varphi \;\big|\; \varphi_1 \,\mathsf{U}\, \varphi_2}$$

eventually

$$\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true}\,\mathsf{U}\,\varphi$$

always

$$\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$$

Examples for LTL formulas:

mutual exclusion: $\Box(\neg \mathit{crit}_1 \vee \neg \mathit{crit}_2)$

railroad-crossing: $\Box(\mathit{train\_is\_near} \rightarrow \mathit{gate\_is\_closed})$

progress property: $\Box(\mathit{request} \rightarrow \Diamond \mathit{response})$

traffic light: $\Box(\mathit{yellow} \vee \bigcirc\neg \mathit{red})$

$$\boxed{\varphi \; ::= \; \textit{true} \;\big|\; a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \neg\varphi \;\big|\; \bigcirc \varphi \;\big|\; \varphi_1 \,\mathsf{U}\, \varphi_2}$$

eventually $\Diamond\varphi \overset{\text{def}}{=} \textit{true}\,\mathsf{U}\,\varphi$

always $\Box\varphi \overset{\text{def}}{=} \neg\Diamond\neg\varphi$

Infinitely often and eventually forever

$$\varphi \ ::= \ \textit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually $\Diamond\varphi \stackrel{\text{def}}{=} \textit{true}\,\mathsf{U}\,\varphi$

always $\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

Infinitely often and eventually forever

$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc\varphi \ \big| \ \varphi_1 \, \mathsf{U} \, \varphi_2$$

eventually $\Diamond\varphi \ \stackrel{\text{def}}{=} \ \mathit{true} \, \mathsf{U} \, \varphi$

always $\Box\varphi \ \stackrel{\text{def}}{=} \ \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

e.g., unconditional fairness $\Box\Diamond \mathit{crit}_i$

strong fairness $\Box\Diamond \mathit{wait}_i \longrightarrow \Box\Diamond \mathit{crit}_i$

Infinitely often and eventually forever

$$\varphi \; ::= \; \mathit{true} \; \big| \; a \; \big| \; \varphi_1 \wedge \varphi_2 \; \big| \; \neg\varphi \; \big| \; \bigcirc \varphi \; \big| \; \varphi_1 \, \mathsf{U} \, \varphi_2$$

eventually $\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true} \, \mathsf{U} \, \varphi$

always $\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

eventually forever $\Diamond\Box\varphi$

e.g.,	unconditional fairness	$\Box\Diamond \mathit{crit}_i$
	strong fairness	$\Box\Diamond \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$
	weak fairness	$\Diamond\Box \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$

LTL-semantics

interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\,A_1\,A_2\,\ldots \in \left(2^{AP}\right)^{\omega}$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$

$\sigma \models a$ iff $A_0 \models a$,i.e., $a \in A_0$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$

$\sigma \models a$ iff $A_0 \models a$,i.e., $a \in A_0$

$\sigma \models \varphi_1 \wedge \varphi_2$ iff $\sigma \models \varphi_1$ and $\sigma \models \varphi_2$

Semantics of LTL over infinite words

for $\sigma = A_0\,A_1\,A_2\,\ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$

$\sigma \models a$ iff $A_0 \models a$,i.e., $a \in A_0$

$\sigma \models \varphi_1 \wedge \varphi_2$ iff $\sigma \models \varphi_1$ and $\sigma \models \varphi_2$

$\sigma \models \neg\varphi$ iff $\sigma \not\models \varphi$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

$$\sigma \models \mathit{true}$$

$$\sigma \models a \quad \text{iff} \quad A_0 \models a \text{ ,i.e., } a \in A_0$$

$$\sigma \models \varphi_1 \wedge \varphi_2 \quad \text{iff} \quad \sigma \models \varphi_1 \text{ and } \sigma \models \varphi_2$$

$$\sigma \models \neg\varphi \quad \text{iff} \quad \sigma \not\models \varphi$$

$$\sigma \models \bigcirc\varphi \quad \text{iff} \quad \mathit{suffix}(\sigma, 1) = A_1\, A_2\, A_3\, \ldots \models \varphi$$

Semantics of LTL over infinite words

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \mathit{true}$		
$\sigma \models a$	iff	$A_0 \models a$,i.e., $a \in A_0$
$\sigma \models \varphi_1 \wedge \varphi_2$	iff	$\sigma \models \varphi_1$ and $\sigma \models \varphi_2$
$\sigma \models \neg\varphi$	iff	$\sigma \not\models \varphi$
$\sigma \models \bigcirc\varphi$	iff	$\mathit{suffix}(\sigma, 1) = A_1\, A_2\, A_3 \ldots \models \varphi$
$\sigma \models \varphi_1 \, \mathsf{U} \, \varphi_2$	iff	there exists $j \geq 0$ such that

$\mathit{suffix}(\sigma, j) = A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2$ and

$\mathit{suffix}(\sigma, i) = A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1$ for $0 \leq i < j$

LT property of LTL formulas

LTLSF3.1-6B

LT property of LTL formulas

interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$

LT property of LTL formulas

interpretation of **LTL** formulas over traces, i.e.,
infinite words over 2^{AP}

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$

LT property of formula φ:

$$Words(\varphi) \stackrel{\text{def}}{=} \left\{\sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi\right\}$$

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$$\vdots$$

$$\begin{array}{lll} \sigma \models \varphi_1 \,\mathsf{U}\, \varphi_2 & \text{iff} & \text{there exists } j \geq 0 \text{ such that} \\ & & A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2 \quad \text{and} \\ & & A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1 \quad \text{for } 0 \leq i < j \end{array}$$

LTL-semantics of derived operators $\Diamond$ and $\Box$

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

$$\vdots$$

$\sigma \models \varphi_1 \,\mathsf{U}\, \varphi_2$ iff there exists $j \geq 0$ such that

$A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2$ and

$A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1$ for $0 \leq i < j$

$\sigma \models \Diamond \varphi$ iff there exists $j \geq 0$ such that

$A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

LTL-semantics of derived operators $\Diamond$ and $\Box$

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^\omega$:

$$
\begin{array}{lll}
 & \vdots & \\
\sigma \models \varphi_1 \,\mathsf{U}\, \varphi_2 & \text{iff} & \text{there exists } j \geq 0 \text{ such that} \\
 & & A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2 \quad \text{and} \\
 & & A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1 \quad \text{for } 0 \leq i < j \\
\sigma \models \Diamond \varphi & \text{iff} & \text{there exists } j \geq 0 \text{ such that} \\
 & & A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi \\
\sigma \models \Box \varphi & \text{iff} & \text{for all } j \geq 0 \text{ we have:} \\
 & & A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi
\end{array}
$$

given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

assumption: $\mathcal{T}$ has no terminal states, i.e.,
all maximal path fragments in $\mathcal{T}$ are infinite

LTL semantics over paths of TS

LTL semantics over paths of TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

LTL semantics over paths of TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

LTL semantics over paths of TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$
$$\text{iff} \quad trace(\pi) \in Words(\varphi)$$

LTL semantics over paths of TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

without terminal states

LTL formula φ over AP

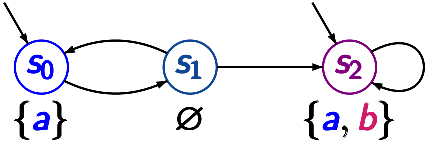
interpretation of φ over infinite path fragments

$$\begin{aligned} \pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad & \text{iff} \quad trace(\pi) \models \varphi \\ & \text{iff} \quad trace(\pi) \in Words(\varphi) \end{aligned}$$

remind: LT property of an LTL formula:

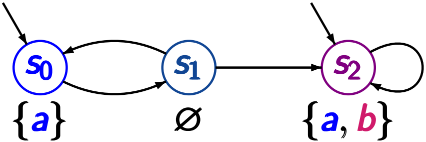
$$Words(\varphi) = \left\{ \sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi \right\}$$

Example: LTL-semantics over paths



$AP = \{a, b\}$

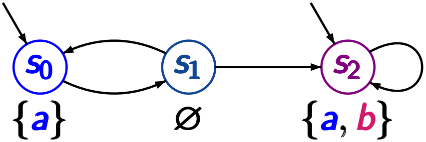
Example: LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$

Example: LTL-semantics over paths

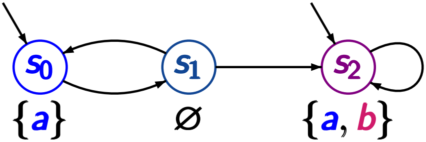


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\} \varnothing \{a, b\}^\omega$

$\pi \models a$

Example: LTL-semantics over paths

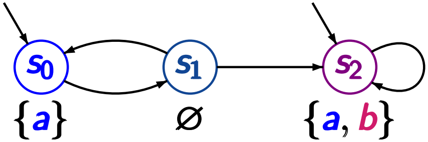


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\quad$ as $L(s_0) = \{a\}$

Example: LTL-semantics over paths



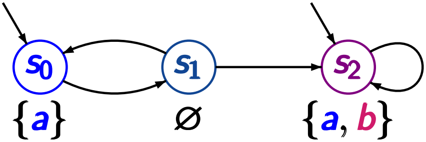
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\qquad$ as $L(s_0) = \{a\}$

$\pi \models \bigcirc(\neg a \wedge \neg b)$

Example: LTL-semantics over paths



$AP = \{a, b\}$

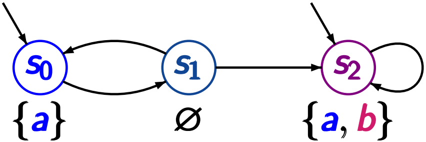
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\quad$ as $L(s_0) = \{a\}$

$\pi \models \bigcirc(\neg a \wedge \neg b)$ $\quad$ as $L(s_1) = \varnothing$

Example: LTL-semantics over paths

LTLSF3.1-9



$AP = \{a, b\}$

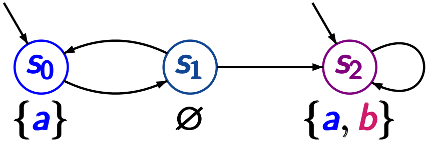
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $\mathit{trace}(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\quad$ as $L(s_0) = \{a\}$

$\pi \models \bigcirc(\neg a \wedge \neg b)$ $\quad$ as $L(s_1) = \varnothing$

$\pi \models \bigcirc \bigcirc (a \wedge b)$

Example: LTL-semantics over paths

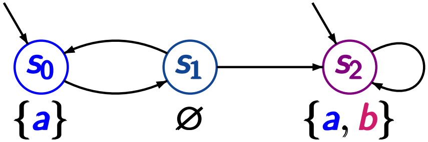


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
$\pi \models \bigcirc \bigcirc (a \wedge b)$	as $L(s_2) = \{a, b\}$

Example: LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $\mathit{trace}(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

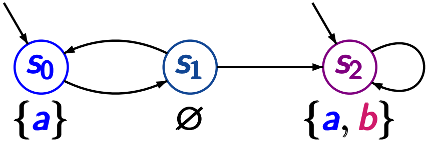
$\pi \models a$, but $\pi \not\models b$ $\qquad$ as $L(s_0) = \{a\}$

$\pi \models \bigcirc(\neg a \wedge \neg b)$ $\qquad$ as $L(s_1) = \varnothing$

$\pi \models \bigcirc \bigcirc (a \wedge b)$ $\qquad$ as $L(s_2) = \{a, b\}$

$\pi \models (\neg b)\, \mathsf{U}\, (a \wedge b)$

Example: LTL-semantics over paths

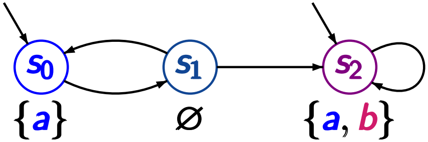


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
$\pi \models \bigcirc\bigcirc(a \wedge b)$	as $L(s_2) = \{a, b\}$
$\pi \models (\neg b) \,\mathsf{U}\, (a \wedge b)$	as $s_0, s_1 \models \neg b$
	and $s_2 \models a \wedge b$

Example: LTL-semantics over paths

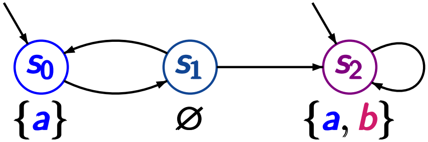


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\,\varnothing\,\{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
$\pi \models \bigcirc\bigcirc(a \wedge b)$	as $L(s_2) = \{a, b\}$
$\pi \models (\neg b)\,\mathsf{U}\,(a \wedge b)$	as $s_0, s_1 \models \neg b$
$\pi \models (\neg b)\,\mathsf{U}\,\Box(a \wedge b)$	and $s_2 \models a \wedge b$

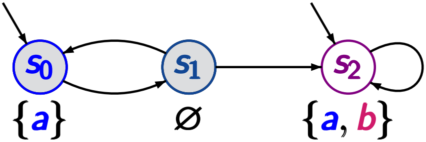
Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1\, \ldots$

Correct or wrong ?

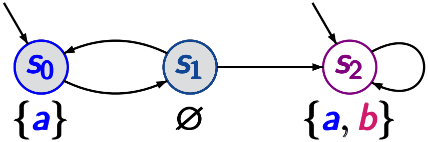


$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1\, \ldots$

$trace(\pi) = (\{a\}\, \varnothing)^\omega$

Correct or wrong ?



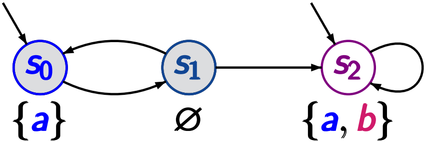
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$

$trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \models a \,\mathsf{U}\, b$?

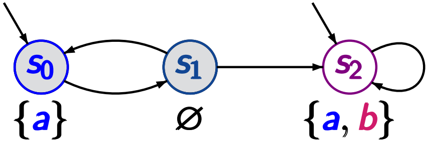
Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1\, \ldots$ $\qquad$ $\mathit{trace}(\pi) = (\{a\}\,\varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$



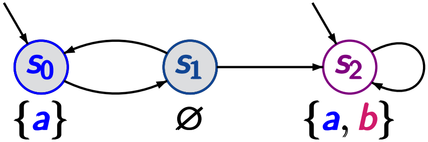
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1\, \ldots$ $\qquad trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$?

Correct or wrong ?



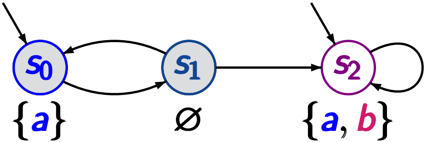
$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

Correct or wrong ?



$AP = \{a, b\}$

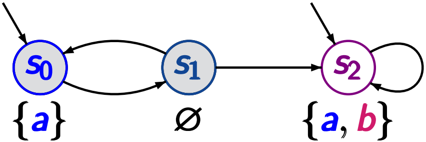
path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \, \ldots$ $\qquad$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$?

Correct or wrong ?



$AP = \{a, b\}$

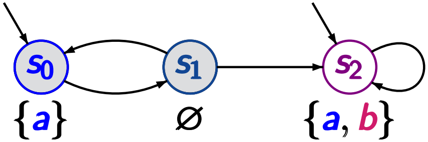
path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $\mathit{trace}(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

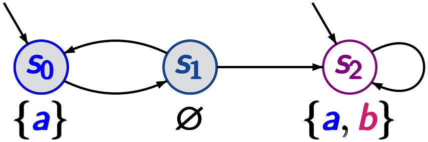
$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \models \Box a$?

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\quad$ $\mathit{trace}(\pi) = (\{a\}\,\varnothing)^\omega$

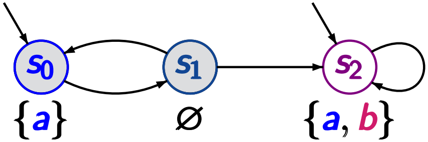
$\pi \not\models a \,\mathsf{U}\, b$ $\quad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\quad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\quad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\quad$ as $s_1 \not\models a$

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\,\varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

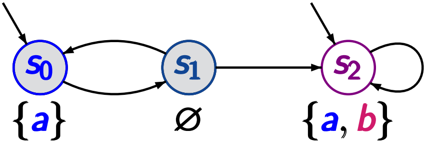
$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$?

Correct or wrong ?



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\, \varnothing)^{\omega}$

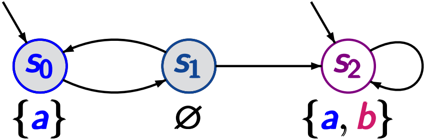
$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$ $\qquad$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1\, \ldots$ $\quad$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$ $\quad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\quad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\quad$ as $s_0 \models \neg b$

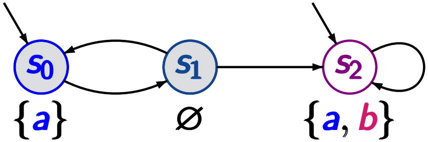
$\pi \not\models \Box a$ $\quad$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$ $\quad$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often

$\pi \models \Diamond\Box a$?

Correct or wrong ?

LTLSF3.1-7



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $\qquad$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

$\pi \not\models a \,\mathsf{U}\, b$	as $s_0 \not\models b$ and $s_1 \not\models a \vee b$
$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$	as $\pi \not\models \Diamond b$
$\pi \models \bigcirc \bigcirc \neg b$	as $s_0 \models \neg b$
$\pi \not\models \Box a$	as $s_1 \not\models a$
$\pi \models \Box\Diamond a$	as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often
$\pi \not\models \Diamond\Box a$	as $\Diamond\Box \mathrel{\widehat{=}}$ eventually forever

LTL-semantics of derived operators

for $\sigma = A_0 \, A_1 \, A_2 \, \ldots \in \left(2^{AP}\right)^{\omega}$:

LTL-semantics of derived operators

for $\sigma = A_0 A_1 A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

LTL-semantics of derived operators

for $\sigma = A_0 A_1 A_2 \ldots \in (2^{AP})^\omega$:

$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Diamond\Box\varphi$	iff	for almost all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$

LTL semantics over the states of a TS

LTL semantics over the states of a TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$s \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s)$$

LTL semantics over the states of a TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$s \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s)$$
$$\quad \text{iff} \quad s \models Words(\varphi)$$

LTL semantics over the states of a TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$\boxed{\begin{aligned} s \models \varphi \quad &\text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s) \\ &\text{iff} \quad s \models Words(\varphi) \end{aligned}}$$

satisfaction relation for LT properties

LTL semantics over the states of a TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$\begin{aligned} s \models \varphi \quad & \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s) \\ & \text{iff} \quad s \models Words(\varphi) \\ & \text{iff} \quad Traces(s) \subseteq Words(\varphi) \end{aligned}$$

Interpretation of LTL formulas over TS

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

$\mathcal{T} \models \varphi$ iff $s_0 \models \varphi$ for all $s_0 \in S_0$

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

$$
\begin{aligned}
\mathcal{T} \models \varphi \quad &\text{iff} \quad s_0 \models \varphi \text{ for all } s_0 \in S_0 \\
&\text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(\mathcal{T})
\end{aligned}
$$

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

$$
\begin{aligned}
\mathcal{T} \models \varphi \quad & \text{iff} \quad s_0 \models \varphi \text{ for all } s_0 \in S_0 \\
& \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(\mathcal{T}) \\
& \text{iff} \quad Traces(\mathcal{T}) \subseteq Words(\varphi)
\end{aligned}
$$

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

$$\begin{array}{lll} \mathcal{T} \models \varphi & \text{iff} & s_0 \models \varphi \text{ for all } s_0 \in S_0 \\ & \text{iff} & trace(\pi) \models \varphi \text{ for all } \pi \in Paths(\mathcal{T}) \\ & \text{iff} & Traces(\mathcal{T}) \subseteq Words(\varphi) \\ & \text{iff} & \mathcal{T} \models Words(\varphi) \end{array}$$

Interpretation of LTL formulas over TS

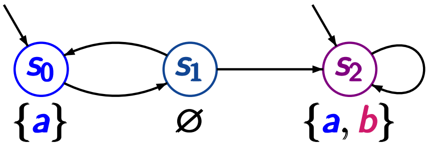
given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

$$
\begin{array}{lll}
\mathcal{T} \models \varphi & \text{iff} & s_0 \models \varphi \text{ for all } s_0 \in S_0 \\
 & \text{iff} & trace(\pi) \models \varphi \text{ for all } \pi \in Paths(\mathcal{T}) \\
 & \text{iff} & Traces(\mathcal{T}) \subseteq Words(\varphi) \\
 & \text{iff} & \mathcal{T} \models Words(\varphi)
\end{array}
$$

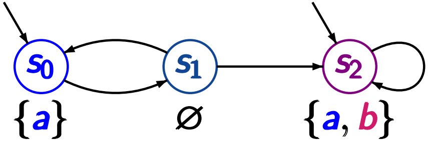
satisfaction relation for LT properties

Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

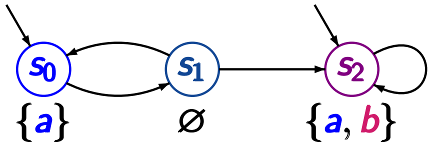
Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

$\mathcal{T} \models a$

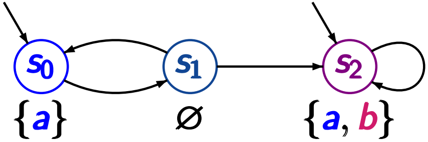
Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

Which formulas hold for $\mathcal{T}$?

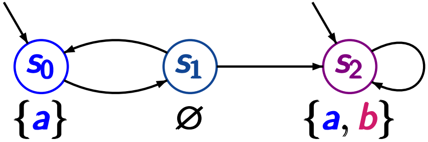


$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \models \Diamond\Box a$

Which formulas hold for $\mathcal{T}$?

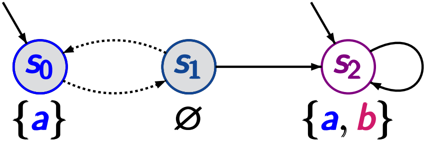


$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$

Which formulas hold for $\mathcal{T}$?

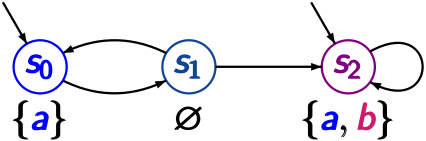


$$AP = \{a, b\}$$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

Which formulas hold for $\mathcal{T}$?



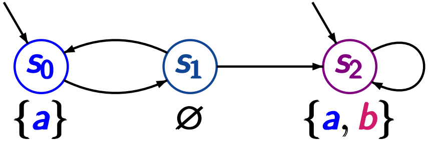
$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

$\mathcal{T} \models \Diamond\Box b \;\vee\; \Box\Diamond(\neg a \wedge \neg b)$

Which formulas hold for $\mathcal{T}$?



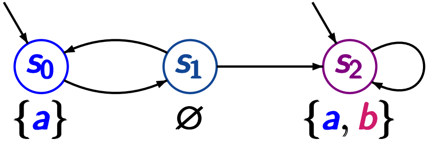
$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

$\mathcal{T} \models \Diamond\Box b \vee \Box\Diamond(\neg a \wedge \neg b)$ as $s_2 \models b$, $s_1 \not\models a, b$

Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

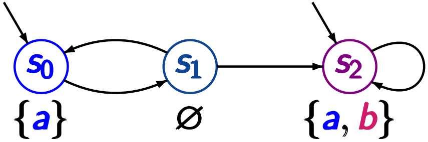
$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

$\mathcal{T} \models \Diamond\Box b \vee \Box\Diamond(\neg a \wedge \neg b)$ as $s_2 \models b$, $s_1 \not\models a, b$

$\mathcal{T} \models \Box(a \rightarrow (\bigcirc\neg a \vee b))$

Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

$\mathcal{T} \models \Diamond\Box b \vee \Box\Diamond(\neg a \wedge \neg b)$ as $s_2 \models b$, $s_1 \not\models a, b$

$\mathcal{T} \models \Box(a \rightarrow (\bigcirc\neg a \vee b))$ as $s_2 \models b$, $s_0 \models \bigcirc\neg a$

Correct or wrong?

For each path π we have: $\pi \models \varphi$ or $\pi \models \neg\varphi$

Correct or wrong?

For each path π we have: $\pi \models \varphi$ or $\pi \models \neg\varphi$

correct, since $\pi \models \neg\varphi$ iff $\pi \not\models \varphi$

Correct or wrong?

For each path π we have: $\pi \models \varphi$ or $\pi \models \neg\varphi$

correct, since $\pi \models \neg\varphi$ iff $\pi \not\models \varphi$

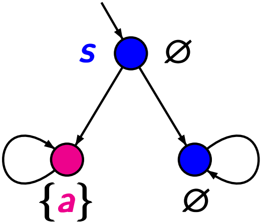
For each state s we have: $s \models \varphi$ or $s \models \neg\varphi$

For each path π we have: $\pi \models \varphi$ or $\pi \models \neg\varphi$

correct, since $\pi \models \neg\varphi$ iff $\pi \not\models \varphi$

For each state s we have: $s \models \varphi$ or $s \models \neg\varphi$

wrong.



$s \not\models \Diamond a$ and $s \not\models \neg\Diamond a$

LTL-formulas for MUTEX protocols

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{\mathsf{wait}_1, \mathsf{crit}_1, \mathsf{wait}_2, \mathsf{crit}_2\}$

- the mutual exclusion property

$$\varphi_{mutex} \;=\; ?$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{\mathsf{wait}_1, \mathsf{crit}_1, \mathsf{wait}_2, \mathsf{crit}_2\}$

- the mutual exclusion property

$$\varphi_{mutex} \; = \; \Box(\neg \mathsf{crit}_1 \vee \neg \mathsf{crit}_2)$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} \;=\; \Box(\neg crit_1 \vee \neg crit_2)$$

- "every process enters the critical section infinitely often"

$$\varphi_{live} \;=\; ?$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} \;=\; \Box(\neg crit_1 \vee \neg crit_2)$$

- "every process enters the critical section infinitely often"

$$\varphi_{live} \;=\; \Box\Diamond crit_1 \;\wedge\; \Box\Diamond crit_2$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} = \Box(\neg crit_1 \vee \neg crit_2)$$

- "every process enters the critical section infinitely often"

$$\varphi_{live} = \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

- starvation freedom
  "every waiting process finally enters its critical section"

$$\varphi_{sf} = ?$$

LTL-formulas for MUTEX protocols

LTL formulas over $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} = \Box(\neg crit_1 \vee \neg crit_2)$$

- "every process enters the critical section infinitely often"

$$\varphi_{live} = \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

- starvation freedom

  "every waiting process finally enters its critical section"

$$\varphi_{sf} = \Box(wait_1 \rightarrow \Diamond crit_1) \wedge \Box(wait_2 \rightarrow \Diamond crit_2)$$

Provide an LTL formula over $AP = \{a, b\}$ for ...

- set of all words $A_0 \, A_1 \, A_2 \, \ldots \in \left(2^{AP}\right)^{\omega}$ such that:

$$\forall i \geq 0. \; (\; a \in A_i \implies i \geq 1 \wedge b \in A_{i-1} \;)$$

- set of all words $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$ such that:

$$\forall i \geq 0.\; (\; a \in A_i \implies i \geq 1 \wedge b \in A_{i-1} \;)$$

$$\forall j \geq 0.\; (\; b \in A_j \;\vee\; a \notin A_{j+1})$$

- set of all words $A_0\,A_1\,A_2 \ldots \in \left(2^{AP}\right)^\omega$ such that:

$$\forall i \geq 0.\ (\ a \in A_i \implies i \geq 1 \wedge b \in A_{i-1}\ )$$

$$\forall j \geq 0.\ (\ b \in A_j \vee a \notin A_{j+1})$$

$$\hat{=}\ \ Words(\ \Box(b \vee \bigcirc\neg a)\ )$$

- set of all words $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^\omega$ such that:

  $$\forall i \geq 0.\ (\ a \in A_i \implies i \geq 1 \wedge b \in A_{i-1}\ )$$

  $$\forall j \geq 0.\ (\ b \in A_j \ \vee\ a \notin A_{j+1})$$

  $$\widehat{=}\ \ Words(\ \Box(b \vee \bigcirc\neg a)\ )$$

- set of all words of the form

  $$\{b\}^{n_1}\,\{a\}\,\{b\}^{n_2}\,\{a\}\,\{b\}^{n_3}\,\{a\}\ldots$$

  where $n_1, n_2, n_3, \ldots \geq 0$

Provide an LTL formula over $AP = \{a, b\}$ for ...

- set of all words $A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^\omega$ such that:

$$\forall i \geq 0.\ (\ a \in A_i \implies i \geq 1 \wedge b \in A_{i-1}\ )$$

$$\forall j \geq 0.\ (\ b \in A_j \ \vee\ a \notin A_{j+1})$$

$$\widehat{=}\quad Words(\ \Box(b \vee \bigcirc\neg a)\ )$$

- set of all words of the form

$$\{b\}^{n_1}\,\{a\}\,\{b\}^{n_2}\,\{a\}\,\{b\}^{n_3}\,\{a\}\ldots$$

where $n_1, n_2, n_3, \ldots \geq 0$

$$\widehat{=}\quad Words(\ \Box((b \wedge \neg a)\ \mathsf{U}\ (a \wedge \neg b))\ )$$

Equivalence of LTL formulas

Equivalence of LTL formulas

$$\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)$$

Equivalence of LTL formulas

$$\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)$$

$$\text{iff} \quad \text{for all transition systems } \mathcal{T}\text{:}$$

$$\mathcal{T} \models \varphi_1 \iff \mathcal{T} \models \varphi_2$$

Equivalence of LTL formulas

$$\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)$$

$$\text{iff} \quad \text{for all transition systems } \mathcal{T}:$$

$$\mathcal{T} \models \varphi_1 \iff \mathcal{T} \models \varphi_2$$

Examples:

$$\begin{array}{lcl} \varphi_1 \vee \varphi_2 & \equiv & \varphi_2 \vee \varphi_1 \\ \neg\neg\varphi & \equiv & \varphi \\ & \vdots & \end{array}$$

all equivalences
from propositional logic

Equivalence of LTL formulas

$$\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)$$

$$\text{iff} \quad \text{for all transition systems } \mathcal{T}:$$

$$\mathcal{T} \models \varphi_1 \iff \mathcal{T} \models \varphi_2$$

Examples:

$$\begin{array}{lcl} \varphi_1 \vee \varphi_2 & \equiv & \varphi_2 \vee \varphi_1 \\ \neg\neg\varphi & \equiv & \varphi \\ & \vdots & \\ \neg\bigcirc\varphi & \equiv & \bigcirc\neg\varphi \end{array}$$

all equivalences
from propositional logic

Self-duality of the next operator

$$\boxed{\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)}$$

Claim: $\neg \bigcirc \varphi \equiv \bigcirc \neg \varphi$ "self-duality of next"

Self-duality of the next operator

$$\boxed{\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)}$$

Claim: $\neg\bigcirc\varphi \equiv \bigcirc\neg\varphi$ "self-duality of next"

Proof: $A_0\, A_1\, A_2\, A_3 \ldots \models \neg\bigcirc\varphi$

Self-duality of the next operator

$$\boxed{\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)}$$

Claim: $\neg \bigcirc \varphi \equiv \bigcirc \neg \varphi$ "self-duality of next"

Proof: $A_0\, A_1\, A_2\, A_3 \ldots \models \neg \bigcirc \varphi$

iff $A_0\, A_1\, A_2\, A_3 \ldots \not\models \bigcirc \varphi$

Self-duality of the next operator

$$\boxed{\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)}$$

Claim: $\neg\bigcirc\varphi \equiv \bigcirc\neg\varphi$ "self-duality of next"

Proof:

$$\begin{array}{lrcl} & A_0\, A_1\, A_2\, A_3 \ldots & \models & \neg\bigcirc\varphi \\ \text{iff} & A_0\, A_1\, A_2\, A_3 \ldots & \not\models & \bigcirc\varphi \\ \text{iff} & A_1\, A_2\, A_3 \ldots & \not\models & \varphi \end{array}$$

Self-duality of the next operator

$$\boxed{\varphi_1 \equiv \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)}$$

Claim: $\neg\bigcirc\varphi \equiv \bigcirc\neg\varphi$ "self-duality of next"

Proof:

$$\begin{array}{lrcl}
 & A_0\, A_1\, A_2\, A_3 \ldots & \models & \neg\bigcirc\varphi \\
\text{iff} & A_0\, A_1\, A_2\, A_3 \ldots & \not\models & \bigcirc\varphi \\
\text{iff} & A_1\, A_2\, A_3 \ldots & \not\models & \varphi \\
\text{iff} & A_1\, A_2\, A_3 \ldots & \models & \neg\varphi
\end{array}$$

Self-duality of the next operator

$$\boxed{\varphi_1 \;\equiv\; \varphi_2 \quad \text{iff} \quad \mathit{Words}(\varphi_1) = \mathit{Words}(\varphi_2)}$$

Claim: $\neg \bigcirc \varphi \;\equiv\; \bigcirc \neg \varphi$ "self-duality of next"

Proof:

$$\begin{array}{lrcl}
 & A_0\,A_1\,A_2\,A_3\ldots & \models & \neg\bigcirc\varphi \\
\text{iff} & A_0\,A_1\,A_2\,A_3\ldots & \not\models & \bigcirc\varphi \\
\text{iff} & A_1\,A_2\,A_3\ldots & \not\models & \varphi \\
\text{iff} & A_1\,A_2\,A_3\ldots & \models & \neg\varphi \\
\text{iff} & A_0\,A_1\,A_2\,A_3\ldots & \models & \bigcirc\neg\varphi
\end{array}$$

Correct or wrong?

$$\Diamond(\varphi \vee \psi) \equiv \Diamond\varphi \vee \Diamond\psi$$

Correct or wrong?

$$\Diamond(\varphi \vee \psi) \;\equiv\; \Diamond\varphi \vee \Diamond\psi$$

correct

Correct or wrong?

$$\Diamond(\varphi \vee \psi) \; \equiv \; \Diamond\varphi \vee \Diamond\psi$$

correct

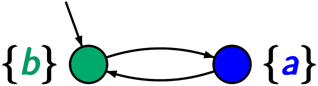
$$\Diamond(\varphi \wedge \psi) \; \equiv \; \Diamond\varphi \wedge \Diamond\psi$$

$$\Diamond(\varphi \vee \psi) \equiv \Diamond\varphi \vee \Diamond\psi$$

correct

$$\Diamond(\varphi \wedge \psi) \equiv \Diamond\varphi \wedge \Diamond\psi$$

wrong,
e.g.,



$\models \Diamond b \wedge \Diamond a$

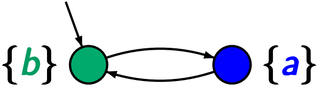
$\not\models \Diamond(b \wedge a)$

$$\Diamond(\varphi \vee \psi) \;\equiv\; \Diamond\varphi \vee \Diamond\psi$$

correct

$$\Diamond(\varphi \wedge \psi) \;\equiv\; \Diamond\varphi \wedge \Diamond\psi$$

wrong,
e.g.,



$\models \Diamond b \wedge \Diamond a$

$\not\models \Diamond(b \wedge a)$

---

similarly: $\Box(\varphi \wedge \psi) \;\equiv\; \Box\varphi \wedge \Box\psi$

$$\Box(\varphi \vee \psi) \;\not\equiv\; \Box\varphi \vee \Box\psi$$

Expansion laws

LTLSF3.1-28

Expansion law for U

until: $\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

Expansion laws for U and $\Diamond$

until: $\varphi \, \mathsf{U} \, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

eventually: $\Diamond\psi \;\equiv\; \psi \;\vee\; \bigcirc\Diamond\psi$

Expansion laws for U and $\Diamond$

until: $\varphi \, \mathsf{U} \, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

eventually: $\Diamond\psi \;\equiv\; \psi \;\vee\; \bigcirc\Diamond\psi$

note: $\Diamond\psi = \mathit{true} \, \mathsf{U} \, \psi$

Expansion laws for U and $\Diamond$

until: $\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

note: $\Diamond\psi = \mathit{true} \,\mathsf{U}\, \psi$

$\equiv \psi \vee (\mathit{true} \wedge \bigcirc(\mathit{true} \,\mathsf{U}\, \psi))$

Expansion laws for U and $\Diamond$

until: $\varphi \, \mathsf{U} \, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

eventually: $\Diamond\psi \;\equiv\; \psi \;\vee\; \bigcirc\Diamond\psi$

note: $\Diamond\psi = \mathit{true} \, \mathsf{U} \, \psi$

$$\equiv \psi \;\vee\; (\mathit{true} \;\wedge\; \bigcirc(\underbrace{\mathit{true} \, \mathsf{U} \, \psi}_{= \Diamond\psi}))$$

Expansion laws for U and $\Diamond$

until: $\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \;\equiv\; \psi \;\vee\; \bigcirc\Diamond\psi$

note: $\Diamond\psi = \mathit{true} \,\mathsf{U}\, \psi$

$$\equiv \psi \;\vee\; (\mathit{true} \;\wedge\; \bigcirc(\underbrace{\mathit{true} \,\mathsf{U}\, \psi}_{=\Diamond\psi}))$$

$$\equiv \psi \;\vee\; \bigcirc\Diamond\psi$$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \;\equiv\; \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \;\equiv\;$ **?**

Expansion laws for U, $\Diamond$ and $\Box$

until:	$\varphi \, \mathsf{U} \, \psi$	$\equiv$	ψ	$\vee$	$(\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$
eventually:	$\Diamond\psi$	$\equiv$	ψ	$\vee$	$\bigcirc\Diamond\psi$
always:	$\Box\psi$	$\equiv$	ψ	$\wedge$	$\bigcirc\Box\psi$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \, \mathsf{U} \, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

eventually: $\Diamond\psi \;\equiv\; \psi \;\vee\; \bigcirc\Diamond\psi$

always: $\Box\psi \;\equiv\; \psi \;\wedge\; \bigcirc\Box\psi$

$$\Box\psi = \neg\Diamond\neg\psi$$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

$$\begin{aligned} \Box\psi &= \neg\Diamond\neg\psi \\ &\equiv \neg(\neg\psi \vee \bigcirc\Diamond\neg\psi) \longleftarrow \boxed{\text{expansion law for } \Diamond} \end{aligned}$$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

$$
\begin{aligned}
\Box\psi &= \neg\Diamond\neg\psi \\
&\equiv \neg(\neg\psi \vee \bigcirc\Diamond\neg\psi) \\
&\equiv \neg\neg\psi \wedge \neg\bigcirc\Diamond\neg\psi \quad \longleftarrow \boxed{\text{de Morgan}}
\end{aligned}
$$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \, \mathsf{U} \, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

$$
\begin{aligned}
\Box\psi &= \neg\Diamond\neg\psi \\
&\equiv \neg(\neg\psi \vee \bigcirc\Diamond\neg\psi) \\
&\equiv \neg\neg\psi \wedge \neg\bigcirc\Diamond\neg\psi \\
&\equiv \psi \wedge \neg\bigcirc\Diamond\neg\psi \quad \longleftarrow \boxed{\text{double negation}}
\end{aligned}
$$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

$$
\begin{aligned}
\Box\psi &= \neg\Diamond\neg\psi \\
&\equiv \neg(\neg\psi \vee \bigcirc\Diamond\neg\psi) \\
&\equiv \neg\neg\psi \wedge \neg\bigcirc\Diamond\neg\psi \\
&\equiv \psi \wedge \bigcirc\neg\Diamond\neg\psi \quad \longleftarrow \boxed{\text{self duality of } \bigcirc}
\end{aligned}
$$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \mathsf{U} \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \mathsf{U} \psi))$

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

$$
\begin{aligned}
\Box\psi &= \neg\Diamond\neg\psi \\
&\equiv \neg(\neg\psi \vee \bigcirc\Diamond\neg\psi) \\
&\equiv \neg\neg\psi \wedge \neg\bigcirc\Diamond\neg\psi \\
&\equiv \psi \wedge \bigcirc\neg\Diamond\neg\psi \\
&\equiv \psi \wedge \bigcirc\Box\psi \qquad \longleftarrow \boxed{\text{definition of } \Box}
\end{aligned}
$$

Expansion laws are fixed point equations

until: $\varphi \mathbf{U} \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \mathbf{U} \psi))$

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

Expansion laws are fixed point equations

until: $\boxed{\varphi \,\mathsf{U}\, \psi} \equiv \psi \vee (\varphi \wedge \bigcirc \boxed{\varphi \,\mathsf{U}\, \psi})$

eventually: $\boxed{\Diamond \psi} \equiv \psi \vee \bigcirc \boxed{\Diamond \psi}$

always: $\boxed{\Box \psi} \equiv \psi \wedge \bigcirc \boxed{\Box \psi}$

Expansion laws are fixed point equations

until: $\boxed{\varphi \, \mathsf{U} \, \psi} \equiv \psi \vee (\varphi \wedge \bigcirc \boxed{\varphi \, \mathsf{U} \, \psi})$

eventually: $\boxed{\Diamond \psi} \equiv \psi \vee \bigcirc \boxed{\Diamond \psi}$

always: $\boxed{\Box \psi} \equiv \psi \wedge \bigcirc \boxed{\Box \psi}$

...don't yield a complete characterization, e.g.,

$$\boxed{\begin{array}{lcl} \mathit{false} & \equiv & a \wedge \bigcirc \mathit{false} \\ \Box a & \equiv & a \wedge \bigcirc \Box a \end{array}}$$

consider $\psi = a$

Expansion laws are fixed point equations

until: $\boxed{\varphi \,\mathsf{U}\, \psi} \equiv \psi \vee (\varphi \wedge \bigcirc \boxed{\varphi \,\mathsf{U}\, \psi})$

eventually: $\boxed{\Diamond\psi} \equiv \psi \vee \bigcirc \boxed{\Diamond\psi}$

always: $\boxed{\Box\psi} \equiv \psi \wedge \bigcirc \boxed{\Box\psi}$

...don't yield a complete characterization, e.g.,

$$\boxed{\begin{array}{lcl} \mathit{false} & \equiv & a \wedge \bigcirc \mathit{false} \\ \Box a & \equiv & a \wedge \bigcirc \Box a \end{array}}$$

although $\Box a \not\equiv \mathit{false}$

Expansion laws are fixed point equations

until: $\varphi \, \mathsf{U} \, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

least fixed point

eventually: $\Diamond \psi \equiv \psi \vee \bigcirc \Diamond \psi$

least fixed point

always: $\Box \psi \equiv \psi \wedge \bigcirc \Box \psi$

...don't yield a complete characterization, e.g.,

$$false \equiv a \wedge \bigcirc false$$
$$\Box a \equiv a \wedge \bigcirc \Box a$$

although $\Box a \not\equiv false$

Expansion laws are fixed point equations

until: $\varphi \mathbf{U} \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \mathbf{U} \psi))$

least fixed point

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

least fixed point

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

greatest fixed point

...don't yield a complete characterization, e.g.,

$$\boxed{\begin{array}{lcl} \mathit{false} & \equiv & a \wedge \bigcirc \mathit{false} \\ \Box a & \equiv & a \wedge \bigcirc\Box a \end{array}}$$

although
$\Box a \not\equiv \mathit{false}$

Expansion law for U

The LTL formula $\chi = \varphi \,\mathbf{U}\, \psi$ is the least solution of

$$\chi \equiv \psi \vee (\varphi \wedge \bigcirc \chi)$$

The LTL formula $\chi = \varphi \,\mathbf{U}\, \psi$ is the least solution of

$$\chi \equiv \psi \vee (\varphi \wedge \bigcirc \chi)$$

i.e., $\mathit{Words}(\varphi \,\mathbf{U}\, \psi)$ least LT-property E s.t.

$$E = \mathit{Words}(\psi) \cup \left\{ A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E \right\}$$

Expansion law for U

The LTL formula $\chi = \varphi \, \mathbf{U} \, \psi$ is the least solution of

$$\chi \equiv \psi \lor (\varphi \land \bigcirc \chi)$$

i.e., $Words(\varphi \, \mathbf{U} \, \psi)$ least LT-property E s.t.

$$E = Words(\psi) \cup \{A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E\}$$

It even holds that $Words(\varphi \, \mathbf{U} \, \psi)$ least LT-property E s.t.

(1) $Words(\psi) \subseteq E$

(2) $\{A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E\} \subseteq E$

The weak until operator W

The weak until operator W

$$\boxed{\varphi \, \mathsf{W} \, \psi \;\stackrel{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi}$$

The weak until operator W

$$\boxed{\varphi \, \mathsf{W} \, \psi \;\stackrel{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi}$$

deriving "always" and "until" from "weak until":

$$\Box\varphi \quad \equiv \quad ?$$

The weak until operator W

$$\boxed{\varphi \, \mathsf{W} \, \psi \ \stackrel{\text{def}}{=} \ (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi}$$

deriving "always" and "until" from "weak until":

$$\Box\varphi \quad \equiv \quad \varphi \, \mathsf{W} \, \mathit{false}$$

The weak until operator W

$$\boxed{\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi}$$

deriving "always" and "until" from "weak until":

$$\Box\varphi \equiv \varphi \,\mathsf{W}\, \textit{false}$$

$$\varphi \,\mathsf{U}\, \psi \equiv \;?$$

The weak until operator W

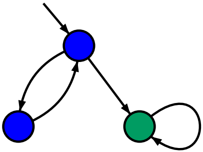
$$\boxed{\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi}$$

deriving "always" and "until" from "weak until":

$$\Box\varphi \equiv \varphi \,\mathsf{W}\, \mathit{false}$$

$$\varphi \,\mathsf{U}\, \psi \equiv (\varphi \,\mathsf{W}\, \psi) \wedge \Diamond\psi$$

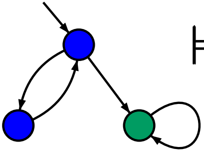
Does $\mathcal{T} \models a \, \mathsf{W} \, b$ hold?



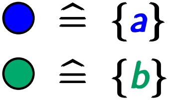
$\bullet \;\widehat{=}\; \{a\}$

$\bullet \;\widehat{=}\; \{b\}$

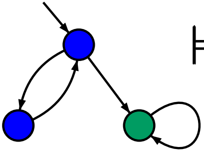
Does $\mathcal{T} \models a \mathsf{W}\, b$ hold?



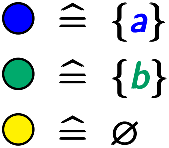
$\models a \mathsf{W}\, b$

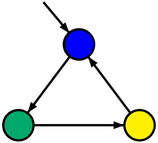


Does $\mathcal{T} \models a \, \mathsf{W} \, b$ hold?

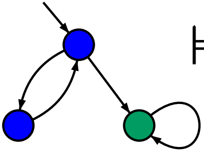


$\models a \, \mathsf{W} \, b$

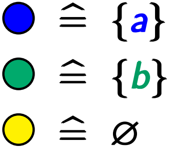


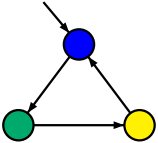


Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?



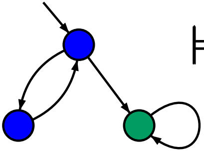
$\models a \,\mathsf{W}\, b$



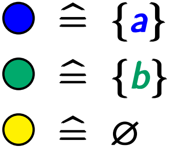


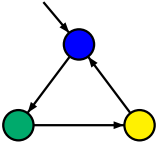
$\models \; a \,\mathsf{W}\, b$ (even $a \,\mathsf{U}\, b$)

Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?

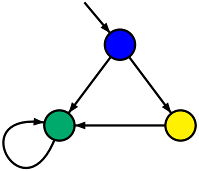


$\models a \,\mathsf{W}\, b$

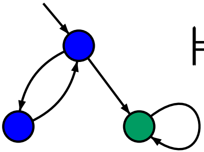




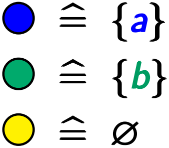
$\models a \,\mathsf{W}\, b$ (even $a \,\mathsf{U}\, b$)

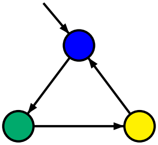


Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?

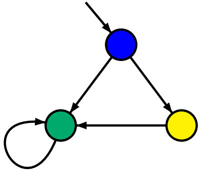


$\models a \,\mathsf{W}\, b$





$\models a \,\mathsf{W}\, b$ (even $a \,\mathsf{U}\, b$)



$\not\models a \,\mathsf{W}\, b$

Duality of U and W

$$\boxed{\varphi \, \mathsf{W} \, \psi \;\overset{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \vee \Box \varphi}$$

goal: express $\neg(\varphi \, \mathsf{U} \, \psi)$ via **W**, and vice versa

Duality of U and W

$$\boxed{\varphi \, \mathsf{W} \, \psi \ \stackrel{\text{def}}{=} \ (\varphi \, \mathsf{U} \, \psi) \ \vee \ \Box\varphi}$$

$$\begin{aligned} & \neg(\varphi \, \mathsf{U} \, \psi) \\ \equiv \ & \big((\varphi \wedge \neg\psi) \ \mathsf{U} \ (\neg\varphi \wedge \neg\psi)\big) \ \vee \ \Box(\varphi \wedge \neg\psi) \end{aligned}$$

Duality of U and W

$$\boxed{\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi}$$

$$\begin{aligned} & \neg(\varphi \,\mathsf{U}\, \psi) \\ \equiv\;& \big((\varphi \wedge \neg\psi) \;\mathsf{U}\; (\neg\varphi \wedge \neg\psi)\big) \vee \Box(\varphi \wedge \neg\psi) \\ \equiv\;& (\varphi \wedge \neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi) \end{aligned}$$

$$\boxed{\varphi \, \mathsf{W} \, \psi \;\overset{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi}$$

$$\begin{aligned}
& \neg(\varphi \, \mathsf{U} \, \psi) \\
\equiv\;& ((\varphi \wedge \neg\psi) \; \mathsf{U} \; (\neg\varphi \wedge \neg\psi)) \vee \Box(\varphi \wedge \neg\psi) \\
\equiv\;& (\varphi \wedge \neg\psi) \, \mathsf{W} \, (\neg\varphi \wedge \neg\psi) \\
\equiv\;& (\neg\psi) \, \mathsf{W} \, (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

Duality of U and W

$$\boxed{\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi}$$

$$\begin{aligned}
& \neg(\varphi \,\mathsf{U}\, \psi) \\
\equiv \;& ((\varphi \wedge \neg\psi) \,\mathsf{U}\, (\neg\varphi \wedge \neg\psi)) \vee \Box(\varphi \wedge \neg\psi) \\
\equiv \;& (\varphi \wedge \neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi) \\
\equiv \;& (\neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

$$\neg(\varphi \,\mathsf{U}\, \psi) \equiv (\neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi)$$

$$\neg(\varphi \,\mathsf{W}\, \psi) \equiv \; ?$$

Duality of U and W

$$\boxed{\varphi \,\mathsf{W}\, \psi \;\overset{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi}$$

$$\begin{aligned}
& \neg(\varphi \,\mathsf{U}\, \psi) \\
\equiv\; & ((\varphi \wedge \neg\psi) \;\mathsf{U}\; (\neg\varphi \wedge \neg\psi)) \vee \Box(\varphi \wedge \neg\psi) \\
\equiv\; & (\varphi \wedge \neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi) \\
\equiv\; & (\neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

$$\boxed{\begin{aligned}
\neg(\varphi \,\mathsf{U}\, \psi) &\equiv (\neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi) \\
\neg(\varphi \,\mathsf{W}\, \psi) &\equiv (\neg\psi) \,\mathsf{U}\, (\neg\varphi \wedge \neg\psi)
\end{aligned}}$$

Expansion laws for U and W

Expansion laws for U and W

$$\varphi \,\mathsf{U}\, \psi \quad \equiv \quad \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

$$\varphi \,\mathsf{W}\, \psi \quad \equiv \quad ?$$

Expansion laws for U and W

$$\varphi \, \mathsf{U} \, \psi \quad \equiv \quad \psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$$

$$\varphi \, \mathsf{W} \, \psi \quad \equiv \quad \psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$$

Expansion laws for U and W

$$\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

smallest solution

$$\varphi \,\mathsf{W}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$$

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

Expansion laws for U and W

$\varphi \mathbf{U} \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \mathbf{U} \psi))$	smallest solution
$\varphi \mathbf{W} \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \mathbf{W} \psi))$	largest solution

$\mathit{Words}(\varphi \mathbf{U} \psi)$ smallest LT-property E s.t.

Expansion laws for U and W

$\varphi \, \mathsf{U} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$	smallest solution
$\varphi \, \mathsf{W} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$	largest solution

$\mathit{Words}(\varphi \, \mathsf{U} \, \psi)$ smallest LT-property E s.t.

$$(1) \quad \mathit{Words}(\psi) \subseteq E$$

$$(2) \quad \{A_0A_1A_2\ldots \in \mathit{Words}(\varphi) : A_1A_2\ldots \in E\} \subseteq E$$

Expansion laws for U and W

$\varphi \mathbin{\mathsf{U}} \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \mathbin{\mathsf{U}} \psi))$	smallest solution
$\varphi \mathbin{\mathsf{W}} \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \mathbin{\mathsf{W}} \psi))$	largest solution

$Words(\varphi \mathbin{\mathsf{U}} \psi)$ smallest LT-property E s.t.

$$(1) \quad Words(\psi) \subseteq E$$

$$(2) \quad \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E$$

$$\Updownarrow$$

$$Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E$$

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

$Words(\varphi \,\mathsf{U}\, \psi)$ smallest LT-property E s.t.

$$Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E$$

Expansion laws for U and W

$\varphi \, \mathsf{U} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$	smallest solution
$\varphi \, \mathsf{W} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$	largest solution

$Words(\varphi \, \mathsf{U} \, \psi)$ smallest LT-property E s.t.

$$Words(\psi) \cup \{A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E\} \subseteq E$$

$Words(\varphi \, \mathsf{W} \, \psi)$ largest LT-property E s.t.

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

$Words(\varphi \,\mathsf{U}\, \psi)$ smallest LT-property E s.t.

$$Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E$$

$Words(\varphi \,\mathsf{W}\, \psi)$ largest LT-property E s.t.

$$Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \supseteq E$$

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

$Words(\varphi \,\mathsf{U}\, \psi)$ smallest LT-property E s.t.

$$Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E$$

$Words(\varphi \,\mathsf{W}\, \psi)$ largest LT-property E s.t.

$$E \subseteq Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\}$$

Expansion laws for U and W

$$\varphi \, \mathsf{U} \, \psi \quad \equiv \quad \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$$

smallest solution

$$\varphi \, \mathsf{W} \, \psi \quad \equiv \quad \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$$

largest solution

Expansion laws for U, W, $\Diamond$, and $\Box$

$$\varphi \, \mathsf{U} \, \psi \quad \equiv \quad \psi \; \vee \; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$$

smallest solution

$$\Diamond \psi \quad \equiv \quad \psi \; \vee \; \bigcirc \Diamond \psi$$

smallest solution

$$\varphi \, \mathsf{W} \, \psi \quad \equiv \quad \psi \; \vee \; (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$$

largest solution

$$\Box \varphi \quad \equiv \quad \varphi \wedge \bigcirc \Box \varphi$$

largest solution

---

remind: $\Diamond \psi \; = \; \mathit{true} \, \mathsf{U} \, \psi$, $\quad \Box \varphi \; \equiv \; \varphi \, \mathsf{W} \, \mathit{false}$